ST. GEORGE'S
CHURCH OF ENGLAND
PRIMARY SCHOOL

MATHS

Theological Underpinning:

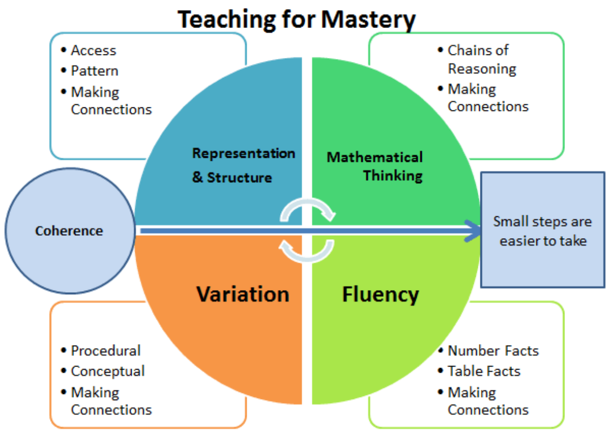
Spiritual growth and development: some of the big questions asked	Biblical references offering insight	Theological underpinning – why is this subject important to us as Christians?
• How can we see the patterns and order in nature? • What provision do we need to make for ourselves, economically and by budgeting well? Why is money important? • In what ways does Maths give us perspective and help us to be good stewards of what we are given? • How does a sense of scale help one's understanding of the world? • How do we actually go about solving problems and what part does evidence play? • What is 'truth' and 'proof'?	• To one man he gave five talents, to another two, and to another one, based on their ability. Then he went on his trip. "The one who received five talents went out at once and invested them and earned five more." Matthew 25:14 • 'God blessed them and said to them, "Be fruitful and increase in number; fill the earth and subdue it. Rule over the fish in the sea and the birds in the sky and over every living creature that moves on the ground." Genesis 1:28 • 'So teach us to count our days, that we may gain a wise heart' Psalm 90 v12	Great is the mystery of faith'. Sometimes mathematics appears to be a series of difficult problems and puzzles. With time, patience, faith and skill we can come to see that the universe is beautifully created, that there is design and order in what originally seemed chaotic. Mathematics shows us that seemingly disconnected things are in fact deeply connected, individual parts of God's one creation. The pattern of God's love is seen in the way balance is restored and difficulty reconciled, just as in Mathematics.

Maths Intent, Implementation and Impact

At St George's we want all children to enjoy learning and exploring maths, become confident mathematicians and use the skills that they learn in their next phase of learning. The language of maths is universal and we have adopted a mastery approach to the teaching and learning of the subject. Due to this universal nature, we place a great importance in making sure that links are made between mathematics and other areas of the curriculum. Using the maths mastery approach, children develop their mathematical fluency without resorting to rote learning and are able to solve non-routine maths problems. We believe that every child can muster a love for maths with the right kind of teaching and support.

To ensure the children achieve this, we use a whole-class approach and spend longer on key units to ensure that children have a secure and deep understanding, thus enabling teaching to build on prior learning throughout their primary school years. The children are encouraged to develop a thorough understanding of the knowledge and skills through a concrete, pictorial, abstract approach to learning. The use of manipulatives enables children to make connections and develop their understanding of maths rather than becoming lost in abstract symbols. As their knowledge, understanding and skills develop, we extend and challenge them to apply their skills into real-life situations through deepening their conceptual understanding. At St George's, we ensure that we meet the needs of children with SEND in the most effective way so that they achieve the best possible outcomes. We want pupils with SEND to acquire the knowledge and skills they need to reach their full potential. As an extremely inclusive school we need them to be ready for the next stage in their education and, ultimately, to succeed in life. To do this, we adapt how we implement the Maths curriculum to meet the needs of pupils with SEND so that we can develop their knowledge, skills and abilities to apply what they know and can do with increasing fluency and independence. Alongside this, children who need further support are given targeted, impactful interventions to support them in succeeding.

To assist our planning, we use the NCETM Primary Mastery Professional Development Materials. These have been built around the core ideas of Maths Mastery (the Big 5). In each of our Maths lessons, we strive to include each aspect of the Big 5 – see image below. The Professional Development Materials strives to allow children to move through the curriculum at the same pace; however, children are expected to work at depth in a concept before moving on to new learning. We aim to achieve this by employing a methodology that encourages not just a procedural proficiency, but rather a conceptual understanding and the flexibility to apply both to problem-solving and reason situations that are unfamiliar.



Coherence
Lessons are broken down into small connected steps that gradually unfold the concept, providing access for all children and leading to a generalisation of the concept and the ability to apply the concept to a range of contexts.

Representation and Structure
Representations used in lessons expose the mathematical structure being taught, the aim being that students can do the maths without recourse to the representation

Mathematical Thinking
If taught ideas are to be understood deeply, they must not merely be passively received but must be worked on by the student: thought about, reasoned with and discussed with others

Fluency
Quick and efficient recall of facts and procedures and the flexibility to move between different contexts and representations of mathematics

Variation
Variation is twofold. It is firstly about how the teacher represents the concept being taught, often in more than one way, to draw attention to critical aspects, and to develop deep and holistic understanding. It is also about the sequencing of the episodes, activities and exercises used within a lesson and follow up practice, paying attention to what is kept the same and what changes, to connect the mathematics and draw attention to mathematical relationships and structure.

In order for the children to 'build [their] fluency in counting, recognising small numbers of items and comparing numbers' we include daily arithmetic from Year 1 to year 6. In Key Stage 1, we focus on mastering number, using the NCETM Mastering Number program to support with the subitising of numbers to ten, as well as recognising number patterns and bonds to all numbers up to 20. The use of the Rekenrek in central to this program allowing children to see the different patterns in their number bonds. Daily arithmetic in Key Stage 2 consists of the repetition of Times Table facts alongside number bond fluency in Years 3 and 4, particularly for the preparation of the Multiplication Test at the end of Year 4. Years 5 and 6 focus more on related number facts that further support the speed and fluency they need to excel in end of Key Stage 2 SATS. To support our pupils with their knowledge of Times Tables, we use Times Tables Rockstars as well as weekly Popcorn Education Timed Multiplication tests where children's scores are collated to capture the progress being made.

The impact of our Mathematics teaching is for the children to be able to:

- Ensure number fluency throughout the school so children are able to apply their number facts
- Make outstanding progress in relation to their individual starting points
- Address misconceptions with confidence
- Explain to others their reasoning and understanding of different mathematical problems using topic specific vocabulary

Maths Whole School Overview

The curriculum is split into a small number of areas called 'spines'. The three spines are *Number, Addition and Subtraction*, *Multiplication and Division*, and *Fractions*. Each spine draws its own conclusion of what children learn.

As well as the spines, small units that include, shape, measure, position and direction, time and angles, are taught discreetly using the White Rose materials and the DfE Ready to Progress documents, still following the mastery approach.

In EYFS we use Mastering number from the NCETM, who have provided small inputs in line with the new EYFS Statutory Guidance.

Non-negotiable key-skills, including times tables tests, are taught weekly.

	Autumn 1	Autumn 2	Spring 1	Spring 2	Summer 1	Summer 2
EYFS **Mastering Number**	See NCETM Mastering Number Scheme					
Year 1	**Transition and baseline assessments** **Spine 1** Basic 1.8 & 1.9 **Shape - 1G1 & 1G2**	**Spine 1** 1.1 1.2 1.3	**Spine 1** 1.4 1.5 1.6	**Spine 1** 1.7 1.10 1.8	**Spine 1** 1.9 **Spine 2** 2.1	**Position and Direction** **Time**
Year 2	**Spine 1** 1.8 1.9 1.11 1.12 Revise 1.7	**Spine 2** 2.2 2.3 2.4 2.5 **Spine 1** 1.13 1.14	**Spine 2** 2.6 **Shape – 2G1** **Spine 1** 1.15 1.16	**Spine 3** Consolidation of 3.0 **Money**	**Measure** **Time** **Consolidation of Spine 1 and Spine 2**	**Position and Direction** **Statistics**
Year 3	**Spine 1** Quick Revise 1.11 1.17 1.18	**Shape – 3G1** **Spine 1** 1.19	**Spine 1** 1.20 **Spine 2**	**Spine 1** 1.21 **Spine 3** 3.1 3.2	**Spine 3** 3.3 3.4	**Spine 2** 2.8 **Shape - 3G2**

			2.7			Time
Year 4	Spine 1 Quick Revise 1.20, 1.21 1.22 1.23	Spine 1 1.24 1.25 Spine 2 2.16 2.9	Spine 2 2.10 2.11 2.13	Spine 3 3.5 Spine 2 2.14 2.15 2.16	Spine 3 3.6 **Symmetry – 4G3** Spine 2 2.12	Spine 2 2.17 **Coordinates – 4G1**
Year 5	Spine 1 Quick Revise 1.23,1.24 1.27 1.26	Spine 2 Quick Revise 2.14, 2.15 2.19 2.29	Spine 2 2.20 2.21	Spine 3 3.7 3.10 **Converting Units**	**Angles – 5G1** Spine 2 2.22	Spine 3 3.8 Spine 1 1.28 1.29
Year 6	Spine 1 **Revise 1.26** (Numbers to 1,000,000) **1.30** (Numbers to 10,000,000) **Revise 1.27** (Negative Numbers) **1.31** (Addition and Subtraction problems) Spine 2 **Revise 2.21** (Factors, Multiples, Prime Numbers) **Revise 2.22/2.28** (Combining addition & subtraction with Multiplication) **2.23** (Multiplication strategies) **2.24** (Dividing strategies) **2.25** (Compensation to calculate) **2.29**	Spine 3 **Revise 3.7** (Equivalent fractions) **Revise 3.5** (Fractions on a line) **3.8** (Adding and subtraction fractions) **3.9** (Multiplying and Dividing fractions) **Converting Units**	Spine 2 **2.27** (Scale factors, ration & proportion) Spine 1 **1.31** (Algebra – problems with two unknowns) Spine 2 **Revise 2.19** (Multiplying and Dividing decimals) Spine 3 **Revise 3.10** (Fractions to Decimals)	Spine 3 **Revise 3.10** (Fractions to Decimals) Spine 2 **Revise 2.16** (Area and Perimeter) **Revise 2.20** (Volume) **2.30** (Area and Perimeter) **Statistics**	**Shape** **Angles** **Position and Direction** **Review and consolidate**	**Review and consolidate**

	(Decimals Multiplication & Division)					

The spines and their corresponding teaching points

Spine 1: Number, Addition and Subtraction

Spine number	Overview	Teaching points
1.1 **Comparison of quantities and measure**	Explore the relationship between numbers and introduce children to the important concept of equivalence; focus on the correct use of comparative language, as well as use of mathematical symbols (<, = and >).	**Teaching Point 1:** Items can be compared according to attributes such as length (or height or breadth), area, volume/capacity or weight/mass. **Teaching Point 2:** When comparing two sets of objects, one set can contain more objects than the other and one set can contain fewer objects than the other, or both sets can contain the same number of objects. **Teaching Point 3:** The symbols <, > and = can be used to express the relative number of objects in two sets, or the relative size of two numbers.
1.2 **Introducing 'whole' and 'parts' (part-part-whole)**	Introduce children to the concept of partitioning, which underpins many of the subsequent segments, and build towards use of the part–part–whole model.	**Teaching point 1:** A 'whole' can be represented by one object; if some of the whole object is missing, it is not the 'whole'. **Teaching point 2:** A whole object can be split into two or more parts in many different ways. The parts might look different; each part will be smaller than the whole, and the parts can be combined to make the whole. **Teaching point 3:** A 'whole' can be represented by a group of discrete objects. If some of the objects in the group are missing, it is not the whole group – it is part of the whole group. **Teaching point 4:** A whole group of objects can be composed of two or more parts and this can be represented using a part–part–whole 'cherry' diagram. The group can be split in many different ways. The parts might look different; each part will be smaller than the whole group and the parts can be combined to make the whole group.
1.3 **Composition of numbers 0-5**	Apply the partitioning structure to the numbers to five, and introduce children to new concepts such as subitising, ordinality and the bar model.	**Teaching point 1:** Numbers can represent how many objects there are in a set; for small sets we can recognise the number of objects (subitise) instead of counting them. **Teaching point 2:** Ordinal numbers indicate a single item or event, rather than a quantity. **Teaching point 3:** Each of the numbers one to five can be partitioned in different ways. **Teaching point 4:** Each of the numbers one to five can be partitioned in a systematic way. **Teaching point 5:** Each of the numbers one to five can be partitioned into two parts; if we know one part, we can find the other part.

		Teaching point 6: The number before a given number is one less; the number after a given number is one more. **Teaching point 7:** Partitioning can be represented using the bar model.
1.4 **Composition of numbers 6-10**	Extend the partitioning structure to the numbers six to ten, explore the five-and-a-bit structure of the numbers, and introduce children to the concept of odd and even numbers.	**Teaching point 1:** The numbers six to nine are composed of 'five and a bit'. Ten is composed of five and five. **Teaching point 2:** Six, seven, eight and nine lie between five and ten on a number line. **Teaching point 3:** Numbers that can be made out of groups of two are even numbers; numbers that can't be made out of groups of two are odd numbers. Even numbers can be partitioned into two odd parts or two even parts; odd numbers can be partitioned into one odd part and one even part. **Teaching point 4:** Each of the numbers six to ten can be partitioned in different ways. The numbers six to ten can be partitioned in a systematic way. **Teaching point 5:** Each of the numbers six to ten can be partitioned into two parts; if we know one part, we can find the other part.
1.5 **Additive structures: introduction to aggregation and partitioning**	Progress to the use of abstract notation (+, − and =) as a way of representing the part–part–whole structure.	**Teaching point 1:** combining two or more parts to make a whole is called aggregation; the addition symbol, +, can be used to represent aggregation. **Teaching point 2:** The equals symbol, =, can be used to show equivalence between the whole and the sum of the parts. **Teaching point 3:** Each addend represents a part, and these are combined to form the whole/sum; we can find the value of the whole by adding the parts. We can represent problems with missing parts using an addition equation with a missing addend. **Teaching point 4:** Breaking a whole down into two or more parts is called partitioning; the subtraction symbol, −, can be used to represent partitioning.
1.6 **Additive structures: introduction to augmentation and reduction**	Introduce children to addition as augmentation, and subtraction as reduction (take away), using a *'first…, then…, now…'* story representation and abstract notation (+, − and =); explore the inverse nature of the two operations.	**Teaching point 1:** An addition context described by a *'first…, then…, now…'* story is an example of augmentation. We can link the story to a numerical representation – each number represents something in the story. **Teaching point 2:** A subtraction context described by a *'first…, then…, now…'* story is an example of reduction. We can link the story to a numerical representation – each number represents something in the story. **Teaching point 3:** Given any two parts of the story we can work out the third part; given any two numbers in the equation we can find the third one. **Teaching point 4:** Addition and subtraction are inverse operations.
1.7 **Addition and subtraction: strategies within 10**	Equip children with a range of useful strategies for addition within ten, including adding and subtracting zero and one, commutativity, adding and subtracting two to/from odd and even numbers, and doubling and halving.	**Teaching point 1**: Addition is commutative: when the order of the addends is changed, the sum remains the same. **Teaching point 2**: Ten can be partitioned into pairs of numbers that sum to ten. Recall of these pairs of numbers supports calculation. **Teaching point 3**: Adding one gives one more; subtracting one gives one less. **Teaching point 4**: Consecutive numbers have a difference of one; we can use this to solve subtraction equations where the subtrahend is one less than the minuend. **Teaching point 5**: Adding two to an odd number gives the next odd number; adding two to an even number gives the next even number. Subtracting two from an odd number gives the previous odd number; subtracting two from an even number gives the previous even number. **Teaching point 6**: Consecutive odd / consecutive even numbers have a difference of two; we can use this to solve subtraction equations where the subtrahend is two less than the minuend. **Teaching point 7**: When zero is added to a number, the number remains unchanged; when zero is subtracted from a number, the number remains unchanged.

		Teaching point 8: Subtracting a number from itself gives a difference of zero. **Teaching point 9**: Doubling a whole number always gives an even number and can be used to add two equal addends; halving is the inverse of doubling and can be used to subtract a number from its double. Memorised doubles/halves can be used to calculate near-doubles/halves. **Teaching point 10**: Addition and subtraction facts for the pairs five and three, and six and three, can be related to known facts and strategies.
1.8 **Composition of numbers: multiples of ten up to 100**	Explore multiples of ten, including counting in tens to 100; apply number facts within ten to addition and subtraction for multiples of ten.	**Teaching point 1**: One ten is equivalent to ten ones. **Teaching point 2**: Multiples of ten can be represented using their names or using numerals. We can count in multiples of ten. **Teaching point 3**: Knowledge of the 0–10 number line can be used to estimate the position of multiples of ten on a 0–100 number line. **Teaching point 4**: Adding ten to a multiple of ten gives the next multiple of ten; subtracting ten from a multiple of ten gives the previous multiple of ten. **Teaching point 5**: Known facts for the numbers *within* ten can be used to add and subtract in multiples of ten by unitising.
1.9 **Composition of numbers: 20-100**	Build on multiples of ten, by introducing non-zero values in the ones place; apply the partitioning structure to these two-digit numbers, decomposing them into tens and ones.	**Teaching point 1**: There is a set counting sequence for counting to 100 and beyond. **Teaching point 2**: Objects can be counted efficiently by making groups of ten. The digits in the numbers 20–99 tell us about their value. **Teaching point 3**: Each number on the 0–100 number line has a unique position. **Teaching point 4**: The relative size of two two-digit numbers can be determined by first examining the tens digits and then, if necessary, examining the ones digits, with reference to the cardinal or ordinal value of the numbers. **Teaching point 5**: Each two-digit number can be partitioned into a tens part and a ones part. **Teaching point 6**: The tens and ones structure of two-digit numbers can be used to support additive calculation.
1.10 **Composition of numbers: 11-19**	Explore the ten-and-a-bit nature of the numbers 11–19, using the partitioning structure; apply number facts within ten to addition and subtraction of single-digit numbers to/from the numbers 11–19.	**Teaching point 1**: The digits in the numbers 11–19 tell us about their value. **Teaching point 2**: The numbers 11–19 can be formed by combining a ten and ones, and can be partitioned into a ten and ones. **Teaching point 3**: A number is even if the ones digit is even; it *can* be made from groups of two. A number is odd if the ones digit is odd; it *can't* be made from groups of two. **Teaching point 4**: Doubling the numbers 6–9 (inclusive) gives an even teen number; halving an even teen number gives a number from six to nine (inclusive). **Teaching point 5**: Addition and subtraction facts within 10 can be applied to addition and subtraction within 20.
1.11 **Addition and subtraction: bridging 10**	Apply the aggregation and augmentation structures of addition to three single-digit numbers, exploring commutativity and associativity, to work towards strategies for adding and subtracting across ten.	**Teaching point 1**: Addition of three addends can be described by an aggregation story with three parts. **Teaching point 2**: Addition of three addends can be described by an augmentation story with a ***'first…, then…, then…, now…'*** structure. **Teaching point 3**: The order in which addends (parts) are added or grouped does not change the sum (associative and commutative laws). **Teaching point 4**: When we are adding three numbers, we choose the most efficient order in which to add them, including identifying two addends that make ten (combining). **Teaching point 5**: We can add two numbers which bridge the tens boundary by using a 'make ten' strategy.

		Teaching point 6: We can subtract across the tens boundary by subtracting *through* ten or subtracting *from* ten.
1.12 **Subtraction as difference**	Introduce children to subtraction as difference, the third and final subtraction structure; review consecutive numbers, as well as consecutive odd/even numbers, in the context of difference.	**Teaching point 1**: Difference compares the number of objects in one set with the number of objects in another set; or the difference between two measures. **Teaching point 2**: Difference is one of the structures of subtraction. **Teaching point 3**: Consecutive whole numbers have a difference of one; consecutive odd/even numbers have a difference of two. **Teaching point 4**: We can apply the structure of difference to compare data.
1.13 **Addition and subtraction: two-digit and single digit numbers**	Build on segments 1.8, 1.9 and 1.10 to equip children with useful strategies for addition and subtraction of a single-digit number to/from two-digit numbers.	**Teaching point 1**: Knowledge of the number line, and quantity values of numbers, can be applied to add/subtract one to/from a given two-digit number. **Teaching point 2**: Known facts for the numbers *within* ten can be applied to addition/subtraction of a single-digit number to/from a two-digit number. **Teaching point 3**: Knowledge of numbers which sum to ten can be applied to the addition of a single-digit number and two-digit number that sum to a multiple of ten, or subtraction of a single-digit number from a multiple of ten. **Teaching point 4**: Known strategies for addition or subtraction bridging ten can be applied to addition or subtraction bridging a multiple of ten.
1.14 **Addition and subtraction: two-digit numbers and multiples of ten**	Explore counting on, and back, in ten from any two-digit number; apply number facts within ten to the addition and subtraction of multiples of ten.	**Teaching point 1**: When finding ten more or ten less than any two-digit number, the ones digit does not change. **Teaching point 2**: When ten is added or subtracted to/from a two-digit number, the tens digit changes and the ones digit stays the same. **Teaching point 3**: Knowledge of number facts within ten can be applied to adding or subtracting multiples of ten to/from a two-digit number. **Teaching point 4**: Two-digit numbers can be partitioned in different ways.
1.15 **Addition: two-digit and two-digit numbers**	Build on segments 1.13 and 1.14 to equip children with useful strategies for addition of two or more two-digit numbers, partitioning two-digit numbers into tens and ones before calculation.	**Teaching point 1**: Known strategies can be combined to add two multiples of ten to two single-digit numbers. **Teaching point 2**: Two two-digit numbers can be added by partitioning one or both of them into tens and one.
1.16 **Subtraction: two-digit and two-digit numbers**	Build on segments 1.13 and 1.14 to equip children with useful strategies for subtraction of one two-digit number from another, partitioning two-digit numbers into tens and ones before calculation.	**Teaching point 1**: Known strategies can be used to subtract a multiple of ten and a single-digit number from a two-digit number. **Teaching point 2**: A two-digit number can be subtracted from a two-digit number by partitioning the subtrahend into tens and ones.

1.17 **Composition and calculation: 100 and bridging 100**	Explore the additive and multiplicative composition of 100; draw on known strategies and number facts to calculate across the 100 boundary.	**Teaching point 1**: There are ten tens in 100; there are 100 ones in 100. 100 can also be composed multiplicatively from 50, 25 or 20, units that are commonly used in graphing and measures. **Teaching point 2**: Known addition facts can be used to calculate complements to 100. **Teaching point 3**: Known strategies for addition and subtraction across the tens boundary can be combined with unitising to count and calculate across the hundreds boundary in multiples of ten. **Teaching point 4**: Knowledge of two-digit numbers can be extended to count and calculate across the hundreds boundary from/to any two-digit number in ones or ten.
1.18 **Composition and calculation: three-digit numbers**	Explore the composition of three-digit numbers; use place-value and partitioning knowledge to support additive calculation, and extend known additive strategies to three-digit numbers.	**Teaching point 1**: Three-digit numbers can be composed additively from hundreds, tens and ones; this structure can be used to support additive calculation. **Teaching point 2**: Each number on the 0 to 1,000 number line has a unique position. **Teaching point 3**: The smallest three-digit number is 100, and the largest three-digit number is 999; the relative size of two three-digit numbers can be determined by examining the hundreds digits, then the tens digits, and then the ones digits, as necessary. **Teaching point 4**: Three-digit multiples of ten can be expressed multiplicatively and additively, in terms of tens or hundreds. **Teaching point 5**: Known facts and strategies for addition and subtraction within and across ten, and within and across 100, can be used to support additive calculation within 1,000. **Teaching point 6**: Familiar counting sequences can be extended up to 1,000.
1.19 **Securing mental strategies: calculation up to 999**	Build on segments 1.15 and 1.16 to equip children with useful calculation strategies for bridging hundreds boundaries, and three-digit numbers; continue to use the partitioning structure to facilitate calculation.	**Teaching point 1**: Known partitioning strategies for adding two-digit numbers within 100 can be extended to the mental addition of two-digit numbers that bridge 100, and addition of three-digit numbers. **Teaching point 2**: Transforming addition calculations into equivalent calculations can support efficient mental strategies. **Teaching point 3**: Subtraction calculations can be solved using a 'finding the difference' strategy; this can be thought of as 'adding on' to find a missing part. **Teaching point 4**: The order of addition and subtraction steps in a multi-step calculation can be chosen or manipulated such as to simplify the arithmetic.
1.20 **Algorithms: column addition**	Introduce children to the column algorithm for addition calculations, applying the algorithm to a variety of aggregation and augmentation contexts for two-digit and three-digit numbers; explore regrouping (column total is ten or greater) in detail.	**Teaching point 1**: Any numbers can be added together using an algorithm called *'column addition'*. **Teaching point 2**: The digits of the addends must be aligned correctly before the algorithm is applied. **Teaching point 3**: In column addition, the digits of the addends are added working from the least significant digit (on the right) to the most significant digit (on the left). **Teaching point 4**: If any column sums to ten or greater, we must *'regroup'*. **Teaching point 5**: The numbers within each column should be added in the most efficient order.
1.21 **Algorithms: column subtraction**	Introduce children to the column algorithm for subtraction calculations, applying the algorithm to a variety of partitioning, reduction and difference contexts for two-digit and three-digit	**Teaching point 1**: One number can be subtracted from another using an algorithm called *'column subtraction'*; the digits of the minuend and subtrahend must be aligned correctly; the algorithm is applied working from the least significant digit (on the right) to the most significant digit (on the left). **Teaching point 2**: If there is an insufficient number of any unit to subtract from in a given column, we must exchange from the column to the left.

	numbers; explore exchange (insufficient quantity to subtract from in a column) in detail.	
1.22 **Composition and calculation: 1000 and four-digit numbers**	Explore the composition of 1,000 and four-digit numbers, using the partitioning structure, and make links to measures; introduce children to calculation across thousands boundaries, and extend column algorithms and rounding to four-digit numbers.	**Teaching point 1**: Ten hundreds make 1,000, which can also be decomposed into 100 tens and 1,000 ones. **Teaching point 2**: When multiples of 100 are added or subtracted, the sum or difference is always a multiple of 100. **Teaching point 3**: Numbers over 1,000 have a structure that relates to their size. This means they can be ordered, composed and decomposed. **Teaching point 4**: Numbers can be rounded to simplify calculations or to indicate approximate sizes. **Teaching point 5**: Calculation approaches learnt for three-digit numbers can be applied to four-digit numbers. **Teaching point 6**: 1,000 can also be composed multiplicatively from 500s, 250s or 200s, units that are commonly used in graphing and measures.
1.23 **Composition and calculation: tenths**	Introduce children to tenths using both the partitioning structure and ideas of place value; apply additive facts and strategies, including column algorithms, and rounding to numbers with tenths.	**Teaching point 1**: When one is divided into ten equal parts, each part is one tenth of the whole. **Teaching point 2**: Tenths can be expressed as decimal fractions; the number written '0.1' is one tenth; one is ten times the size of 0.1. **Teaching point 3**: We can count in tenths up to and beyond one. **Teaching point 4**: Numbers with tenths can be composed additively and multiplicatively. **Teaching point 5**: Known facts and strategies, including column algorithms, can be applied to calculations for numbers with tenths. **Teaching point 6**: Numbers with tenths can be rounded to the nearest whole number by examining the value of the tenths digit.
1.24 **Composition and calculation: hundredths and thousandths**	Building on segment 1.23, introduce children to hundredths (and thousandths) using both the partitioning structure and ideas of place value; apply additive facts and strategies, including column algorithms, and rounding to numbers with hundredths (and thousandths).	**Teaching point 1**: When one is divided into 100 equal parts, each part is one hundredth of the whole. When one tenth of a whole is divided into ten equal parts, each part is one hundredth of the whole. **Teaching point 2**: Hundredths can be expressed as decimal fractions; the number written '0.01' is one hundredth; one is one hundred times the size of 0.01; 0.1 is ten times the size of 0.01. **Teaching point 3**: We can count in hundredths up to and beyond one. **Teaching point 4**: Numbers with hundredths can be composed additively and multiplicatively. **Teaching point 5**: Numbers with tenths and hundredths are commonly used in measurement, scales and graphing contexts. **Teaching point 6**: Known facts and strategies, including column algorithms, can be applied to calculations for numbers with hundredths; the same approaches can be used for numbers with hundredths as are used for numbers with tenths. **Teaching point 7**: Numbers with hundredths can be rounded to the nearest tenth by examining the value of the hundredths digit or to the nearest whole number by examining the value of the tenths digit. **Teaching point 8**: When one is divided into 1,000 equal parts, each part is one thousandth of the whole. Knowledge and strategies for numbers with tenths and hundredths can be applied to numbers with thousandths.
1.25	Building on segments 1.23 and 1.24, introduce children to conventions for	

Addition and subtraction: money	expressing monetary value and explore the equivalence of 100 p and £1; encourage children to select column algorithms or equivalent calculations where most appropriate.	**Teaching point 1**: One penny is one hundredth of a pound; conventions for expressing quantities of money are based on expressing numbers with tenths and hundredths. **Teaching point 2**: Equivalent calculation strategies for addition can be used to efficiently add commonly-used prices. **Teaching point 3**: The 'working forwards'/'finding the difference' strategy for subtraction is an efficient way to calculate the change due when paying in whole pounds or notes. **Teaching point 4**: Column methods can be used to add and subtract quantities of money. **Teaching point 5**: Finding change when purchasing several items uses the part–part–(part–)whole structure.
1.26 **Composition and calculation: multiples of 1000 up to 1, 000, 000**	Explore the composition of six-digit, whole-thousand numbers, using the partitioning structure; apply knowledge and strategies from segments 1.17 and 1.18 combined with unitising in 1,000s, as well as column methods and rounding.	**Teaching point 1**: Understanding of numbers composed of hundred thousands, ten thousands and one thousands can be supported by making links to numbers composed of hundreds, tens and ones. **Teaching point 2**: Multiples of 1,000 up to 1,000,000 can be placed in the linear number system by drawing on knowledge of the place of numbers up to 1,000 in the linear number system. **Teaching point 3**: Numbers can be ordered and compared using knowledge of their composition and of their place in the linear number system. **Teaching point 4**: Calculation approaches for numbers up to 1,000 can be applied to multiples of 1,000 up to 1,000,000. **Teaching point 5**: Numbers can be rounded to simplify calculations or to indicate approximate sizes. **Teaching point 6**: Known patterns can be used to divide 10,000 and 100,000 into two, four and five equal parts. These units are commonly used in graphing and measures.
1.27 **Negative numbers: counting, comparing and calculating**	Introduce children to negative numbers, making links to everyday contexts; explore addition and subtraction below zero and across zero.	**Teaching point 1**: Positive and negative numbers can be used to represent change. **Teaching point 2**: Our number system includes numbers that are less than zero; these are negative numbers. Numbers greater than zero are positive numbers. **Teaching point 3**: The negative/minus symbol (−) is placed before a numeral to indicate that the value is a negative number. **Teaching point 4**: Negative numbers can be shown on horizontal scales; numbers to the left of zero are negative (less than zero) and numbers to the right of zero are positive (greater than zero). The larger the value of the numeral after the negative/minus symbol, the further the number is from zero. **Teaching point 5**: Knowledge of the positions of positive and negative numbers in the number system can be used to calculate intervals across zero. **Teaching point 6**: Negative numbers are used in coordinate and graphing contexts.
1.28 **Common structures and the part-part-whole relationship**	Extend the part–part–part–whole structure (three or more parts) to solve missing part/whole problems in a range of contexts; draw on number composition and additive concepts from across the spine, focusing on the structural equivalence of the problems.	**Teaching point 1**: Mathematical relationships encountered at primary level are either additive or multiplicative; both of these can be observed within the structure of part–part–whole relationships. **Teaching point 2**: Problems in many different contexts can be solved by adding together the parts to find the whole. Different strategies can be used to calculate the whole, but the structure of the problem remains the same. **Teaching point 3**: If the value of the whole is known, along with the values of all but one of the parts, the value of the missing part can be calculated. Different strategies can be used to calculate the missing part, but the structure of the problem remains the same. **Teaching point 4**: Problems in many different contexts have the 'missing-part' structure.

1.29 **Using equivalence and the compensation property to calculate**	Explore the effect on the sum of changing the value of one or both addends; explore the effect on the difference of changing the value of the minuend, the subtrahend or both. Apply knowledge of compensation properties and inverse operations to calculate and balance equations.	**Teaching point 1**: If one addend is increased and the other is decreased by the same amount, the sum stays the same. (same sum) **Teaching point 2**: If one addend is increased (or decreased) and the other is kept the same, the sum increases (or decreases) by the same amount. **Teaching point 3**: If the minuend and subtrahend are changed by the same amount, the difference stays the same. (same difference) **Teaching point 4**: If the minuend is increased (or decreased) and the subtrahend is kept the same, the difference increases (or decreases) by the same amount. **Teaching point 5**: If the minuend is kept the same and the subtrahend is increased (or decreased), the difference decreases (or increases) by the same amount. **Teaching point 6**: The value of the expressions on each side of an equals symbol must be the same; addition and subtraction are inverse operations. We can use this knowledge to balance equations and solve problems.
1.30 **Composition and calculation: numbers up to 10, 000, 000**	Building on segment 1.26, explore six-digit numbers that are not whole thousands, and then extend to seven-digit numbers; apply additive facts and strategies, including column algorithms, and rounding to these numbers.	**Teaching point 1**: Patterns seen in other powers of ten can be extended to the unit 1,000,000. **Teaching point 2**: Seven-digit numbers can be written, read and ordered by identifying the number of millions, the number of thousands and the number of hundreds, tens and ones. **Teaching point 3**: The digits in a number indicate its structure so it can be composed and decomposed. **Teaching point 4**: Knowledge of crossing thousands boundaries can be used to work to and across millions boundaries. **Teaching point 5**: Sometimes numbers are rounded as approximations to eliminate an unnecessary level of detail; rounded numbers are also used to give an estimate or average. At other times, precise readings are useful. **Teaching point 6**: Fluent calculation requires the flexibility to move between mental and written methods according to the specific numbers in a calculation.
1.31 **Problems with two unknowns**	Equip children with strategies for solving problems with two unknowns, including using the bar model to represent relationships between known numbers, and working systematically.	**Teaching point 1**: Problems with two unknowns can have one solution or more than one solution (or no solution). A relationship between the two unknowns can be described in different ways, including additively and multiplicatively. **Teaching point 2**: Model drawing can be used to expose the structure of problems with two unknowns. **Teaching point 3**: A problem with two unknowns has only one solution if the sum of the two unknowns and the difference between them is given (*'sum-and-difference problems'*) or if the sum of the two unknowns and a multiplicative relationship between them is given (*'sum-and-multiple problems'*). **Teaching point 4**: Other problems with two unknowns have only one solution. **Teaching point 5**: Some problems with two unknowns can't easily be solved using model drawing but can be solved by a 'trial-and-improvement' approach; these problems may have one solution, several solutions or an infinite number of solutions.

Spine 2: Multiplication and division

Spine number	Overview	Teaching points
2.1 **Counting, unitising and coins**	Explore the concept of unitising by counting in units of two, five or ten; investigate how objects can be counted efficiently by counting in units other than one; apply unitising in the context of the low-denomination coins (1 p, 2 p, 5 p and 10 p).	**Teaching point 1**: We can count efficiently by counting in groups of two. **Teaching point 2**: We can count efficiently by counting in groups of ten. **Teaching point 3**: We can count efficiently by counting in groups of five. **Teaching point 4**: A coin has a value which is independent of its size, shape, colour or mass. **Teaching point 5**: The *number* of coins in a set is different from the *value* of the coins in a set; knowledge of counting in groups of two, five or ten can be used to work out the value of a set of identical low-denomination coins. **Teaching point 6**: Knowledge of counting in groups of two, five or ten can be used to work out how many identical low-denomination coins are needed to make a given value.
2.2 **Structures: multiplication representing equal groups**	Explore how objects can be arranged in equal groups, and how the number of groups and the size of the groups can be described; represent equally grouped objects with addition and multiplication expressions, connecting multiplication to repeated addition.	**Teaching point 1**: Objects can be grouped into equal or unequal groups. **Teaching point 2**: When describing equally grouped objects, the number of groups and the size of the groups must both be defined. **Teaching point 3**: Equal groups can be represented with a repeated addition expression. **Teaching point 4**: Equal groups can be represented with a multiplication expression. **Teaching point 5**: Multiplication expressions can be written for cases where the groups each contain zero items, and for cases where the groups each contain one item.

2.3 **Times tables: groups of 2 and commutativity (part 1)**	Build up the two times table by combining children's experience of counting in units of two and of representing equal groups; explore how, in a multiplication equation, the factors can appear in either order and the product remains the same.	**Teaching point 1**: For equally grouped objects, the number of groups is a factor, the group size is a factor, and the overall number of objects is the product; this can be represented with a multiplication equation. Counting in multiples of two can be used to find the product when the group size is two. **Teaching point 2**: Counting in multiples of two can be represented by the two times table. Adjacent multiples of two have a difference of two. Facts from the two times table can be used to solve problems about groups of two. **Teaching point 3**: Factor pairs can be written in either order, with the product remaining the same (commutativity).
2.4 **Times tables: groups of 10 and of 5, and factors of 0 and 1**	Build up the ten and five times tables, combining children's experience of counting in units of five or ten and of representing equal groups; explore patterns in the ten and five times tables, and generalise about the product when one factor is zero or one.	**Teaching point 1**: Counting in multiples of ten can be represented by the ten times table. Adjacent multiples of ten have a difference of ten. Facts from the ten times table can be used to solve problems about groups of ten. **Teaching point 2**: Counting in multiples of five can be represented by the five times table. Adjacent multiples of five have a difference of five. Facts from the five times table can be used to solve problems about groups of five. **Teaching point 3**: Skip counting and grouping can be used to explore the relationship between the five times table and the ten times table. **Teaching point 4**: When zero is a factor, the product is zero. When one is a factor, the product is equal to the other factor (if there are only two factors.
2.5 **Commutativity (part 2): doubling and halving**	Explore how one multiplication equation can have two different grouping interpretations (e.g., an equation from the two times table can be interpreted in terms of groups of two, or two equal groups); make connections between the two times table, doubling and halving.	**Teaching point 1**: The same multiplication equation can have two different grouping interpretations. Problems about two/five/ten equal groups can be solved using facts from the two/five/ten times table. (commutativity) **Teaching point 2**: If two is a factor, knowledge of doubling facts can be used to find the product; problems about doubling can be solved using facts from the two times table. **Teaching point 3**: Halving is the inverse of doubling; problems about halving can be solved using facts from the two times table and known doubling facts. **Teaching point 4**: Products in the ten times table are double the products in the five times table; products in the five times table are half of the products in the ten times table.
2.6 **Structures: quotitive and partitive division**	Introduce the quotitive and partitive structures of division; skip count using the divisor, or use known multiplication facts, to find the quotient; generalise about the quotient when dividend = 0, dividend = divisor, or divisor = 1.	**Teaching point 1**: Objects can be grouped equally, sometimes with a remainder. **Teaching point 2**: Division equations can be used to represent 'grouping' problems, where the total quantity (dividend) and the group size (divisor) are known; the number of groups (quotient) can be calculated by skip counting in the divisor. (quotitive division) **Teaching point 3**: Division equations can be used to represent 'sharing' problems, where the total quantity (dividend) and the number we are sharing between (divisor) are known; the size of the shares (quotient) can be calculated by skip counting in the divisor. (partitive division) **Teaching point 4**: Strategies for finding the quotient, that are more efficient than skip counting, include using known multiplication facts and, when the divisor is two, using known halving facts. **Teaching point 5**: When the dividend is zero, the quotient is zero; when the dividend is equal to the divisor, the quotient is one; when the divisor is equal to one, the quotient is equal to the dividend.
2.7 **Times tables: 2, 4 and 8, and the relationship between them**	Build up the four/eight times table; using different structures/interpretations of multiplication and division, solve problems related to these tables; explore connections between the two, four and eight times tables.	**Teaching point 1**: Counting in multiples of four can be represented by the four times table. Adjacent multiples of four have a difference of four. Facts from the four times table can be used to solve multiplication and division problems with different structures. **Teaching point 2**: Products in the four times table are double the products in the two times table; products in the two times table are half of the products in the four times table.

		Teaching point 3: Counting in multiples of eight can be represented by the eight times table. Adjacent multiples of eight have a difference of eight. Facts from the eight times table can be used to solve multiplication and division problems with different structures. **Teaching point 4**: Products in the eight times table are double the products in the four times table; products in the four times table are half of the products in the eight times table. Products that are in the two, four and eight times tables share the same factors. **Teaching point 5**: Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by two, four or eight.
2.8 **Times tables: 3, 6 and 9, and the relationship between them**	Build up the three/six/nine times table; using different structures/interpretations of multiplication and division, solve problems related to these tables; explore connections between the three, six and nine times tables.	**Teaching point 1**: Counting in multiples of three can be represented by the three times table. Adjacent multiples of three have a difference of three. Facts from the three times table can be used to solve multiplication and division problems with different structures. **Teaching point 2**: Counting in multiples of six can be represented by the six times table. Adjacent multiples of six have a difference of six. Facts from the six times table can be used to solve multiplication and division problems with different structures. **Teaching point 3**: Products in the six times table are double the products in the three times table; products in the three times table are half of the products in the six times table. **Teaching point 4**: Counting in multiples of nine can be represented by the nine times table. Adjacent multiples of nine have a difference of nine. Facts from the nine times table can be used to solve multiplication and division problems with different structures. **Teaching point 5**: Products in the nine times table are triple the products in the three times table. Products that are in the three, six and nine times tables share the same factors. **Teaching point 6**: Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by three, six or nine.
2.9 **Times tables: 7 and patterns within/across times tables**	Build up the seven times table and solve associated multiplication and division problems; explore times table patterns including generalising about the product in terms of odd/even factors, reviewing divisibility rules, and exploring square numbers.	**Teaching point 1**: Counting in multiples of seven can be represented by the seven times table. Adjacent multiples of seven have a difference of seven. Facts from the seven times table can be used to solve multiplication and division problems with different structures. **Teaching point 2**: When both factors are odd numbers, the product is an odd number; when one factor is an odd number and the other is an even number, the product is an even number; when both factors are even numbers, the product is an even number. **Teaching point 3**: When both factors have the same value, the product is called a square number; square numbers can be represented by objects arranged in square arrays. **Teaching point 4**: Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by particular divisors.
2.10 **Connecting multiplication and division, and the distributive law**	Explore why multiplication is commutative while division is not. Build on understanding of the difference between adjacent multiples to explore the distributive law, and apply it to derive multiplication facts.	**Teaching point 1**: Multiplication is commutative; division is not commutative. **Teaching point 2**: Multiplication is distributive: multiplication facts can be derived from related known facts by partitioning one of the factors, and this can be interpreted as partitioning the number of groups; two-part problems that involve addition/subtraction of products with a common factor can be efficiently solved by applying the distributive law. **Teaching point 3**: The distributive law can be used to derive multiplication facts beyond known times tables.
2.11 **Times tables: 11 and 12**	Build up the eleven and twelve times tables using the distributive law, and solve associated multiplication and division problems. Combine known six times table facts with doubling facts and strategies to multiply by twelve.	**Teaching point 1**: The distributive law can be used to build up the 11 times table by partitioning 11 into 10 and 1. Adjacent multiples of 11 have a difference of 11. **Teaching point 2**: The distributive law can be used to build up the 12 times table by partitioning 12 into 10 and 2. Adjacent multiples of 12 have a difference of 12.

		Teaching point 3: Products in the 12 times table are double the products in the six times table; products in the six times table are half of the products in the 12 times table. **Teaching point 4**: Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by 11 or 12.
2.12 **Division with remainders**	Explore how some quantities can be split into equal groups with a remainder, and express this using mathematical notation; practise interpreting the meaning of the remainder in different contexts.	**Teaching point 1**: Objects can be divided into equal groups, sometimes with a remainder; objects can be shared equally, sometimes with a remainder; a remainder can be represented as part of a division equation. **Teaching point 2**: If the dividend *is* a multiple of the divisor, there is *no* remainder; if the dividend *is not* a multiple of the divisor, there *is* a remainder. The remainder is always less than the divisor. **Teaching point 3**: When solving contextual problems involving remainders, the answer to a division calculation must be interpreted carefully to determine how to make sense of the remainder.
2.13 **Calculation: multiplying and dividing by 10 or 100**	Use place-value knowledge to develop strategies for multiplying/dividing by 10 and 100. Generalise about the product or quotient when a factor or the dividend is made 10 or 100 times bigger/smaller.	**Teaching point 1**: Finding 10 times as many is the same as multiplying by 10 (for positive numbers); to multiply a whole number by 10, place a zero after the final digit of that number. **Teaching point 2**: To divide a multiple of 10 by 10, remove the final zero digit (in the ones place) from that number. **Teaching point 3**: Finding 100 times as many is the same as multiplying by 100 (for positive numbers); to multiply a whole number by 100, place two zeros after the final digit of that number. **Teaching point 4**: To divide a multiple of 100 by 100, remove the final two zero digits (in the tens and ones places) from that number. **Teaching point 5**: Multiplying a number by 100 is equivalent to multiplying by 10, and then multiplying the product by 10. Dividing a multiple of 100 by 100 is equivalent to dividing by 10, and then dividing the quotient by 10. **Teaching point 6**: If one factor is made 10 times the size, the product will be 10 times the size. If the dividend is made 10 times the size, the quotient will be 10 times the size. **Teaching point 7**: If one factor is made 100 times the size, the product will be 100 times the size. If the dividend is made 100 times the size, the quotient will be 100 times the size.
2.14 **Multiplication: partitioning leading to short multiplication**	Introduce the short multiplication algorithm, using it to multiply two-/three-digit numbers by single-digit numbers; explore regrouping where necessary.	**Teaching point 1**: The distributive law can be applied to multiply any two-digit number by a single-digit number, by partitioning the two-digit number into tens and ones, multiplying the parts by the single-digit number, then adding the partial products. **Teaching point 2**: Any two-digit number can be multiplied by a single-digit number using an algorithm called *'short multiplication'*; the digits of the factors must be aligned correctly; the algorithm is applied working from the least significant digit (on the right) to the most significant digit (on the left); if the product in any column is ten or greater, we must *'regroup'*. **Teaching point 3**: The distributive law can be applied to multiply any three-digit number by a single-digit number, by partitioning the three-digit number into hundreds, tens and ones, multiplying the parts by the single-digit number, then adding the partial products. **Teaching point 4**: Any three-digit number can be multiplied by a single-digit number using the short multiplication algorithm.
2.15 **Division: partitioning leading to short division**	Introduce the short division algorithm, using it to divide two-/three-digit numbers by single-digit numbers; explore exchange where necessary.	**Teaching point 1**: Any two-digit number can be divided by a single-digit number, by partitioning the two-digit number into tens and ones, dividing the parts by the single-digit number, then adding the partial quotients; if dividing the tens gives a remainder of one or more tens, we must exchange the remaining tens for ones before dividing the resulting ones value by the single-digit number. **Teaching point 2**: Any two-digit number can be divided by a single-digit number using an algorithm called *'short division'*; the algorithm is applied working from the most significant digit (on the left) to the least significant digit (on the right); if there is a remainder in the tens column, we

		must *'exchange'*. **Teaching point 3**: Any three-digit number can be divided by a single-digit number, by partitioning the two-digit number into hundreds, tens and ones, dividing the parts by the single-digit number, then adding the partial quotients; if dividing the hundreds gives a remainder of one or more hundreds, we must exchange the remaining hundreds for tens before dividing the resulting tens value by the single-digit number. **Teaching point 4**: Any three-digit number can be divided by a single-digit number using the short-division algorithm.
2.16 **Multiplicative contexts: area and perimeter 1**	Use addition and multiplication to solve problems about the perimeter of irregular and regular 2D shapes, and to find the area of rectilinear and composite rectilinear shapes; use division to solve associated inverse problems.	**Teaching point 1**: Perimeter is the distance around the edge of a two-dimensional (2D) shape. **Teaching point 2**: Perimeter is measured in units of length and can be calculated by adding together the lengths of the sides of a 2D shape. **Teaching point 3**: Multiplication can be used to calculate the perimeter of a regular polygon; when the perimeter is known, side-lengths can be calculated using division. **Teaching point 4**: Area is the measurement of the surface of a flat item. **Teaching point 5**: Area is measured in square units, such as square centimetres (cm^2) and square metres (m^2). **Teaching point 6**: The area of a rectangle can be calculated using multiplication; the area of a composite rectilinear shape can be found by splitting the shape into smaller rectangles.
2.17 **Structures: using measures and comparison to understand scaling**	Build on segment 2.13 to introduce the scaling structure of multiplication and division; use known multiplication and division strategies to solve problems about scaling/comparison problems.	**Teaching point 1**: A longer length can be described in terms of a shorter length using the language of 'times'; the longer length can be calculated, if the shorter length is known, using multiplication. **Teaching point 2**: A shorter length can be described in terms of a longer length using the language of fractions; the shorter length can be calculated, if the longer length is known, using division. **Teaching point 3**: Other measures can be compared using the language of 'times' and fractions, and calculated using multiplication or division.
2.18 **Using equivalence to calculate**	Develop efficiency in calculation by using equivalence, through adjusting the factors (in multiplication) and the dividend and divisor (in division).	**Teaching point 1**: For multiplication, if there is a multiplicative *increase* to one factor and a corresponding *decrease* to the other factor, the product stays the same. **Teaching point 2**: For division, if there is a multiplicative change to the dividend and a corresponding change to the divisor, the quotient stays the same.
2.19 **Calculation: multiplying and dividing decimal fractions by whole numbers**	Develop strategies for multiplying and dividing decimal fractions by whole numbers, including combining known facts with unitising, multiplying and dividing by 10 and 100, and using adjusting strategies.	**Teaching point 1**: Decimal fractions (with a whole number of tenths or hundredths) can be multiplied by a whole number by using known multiplication facts and unitising. **Teaching point 2**: Multiplying by 0.1 is equivalent to dividing by 10; multiplying by 0.01 is equivalent to dividing by 100. Understanding of place value can be used to divide a number by 10/100: when a number is divided by 10, the digits move one place to the right; when a number is divided by 100, the digits move two places to the right. **Teaching point 3**: To multiply a single-digit number by a decimal fraction with up to two decimal places, convert the decimal fraction to an integer by multiplying by 10 or 100, perform the resulting calculation using an appropriate strategy, then adjust the product by dividing by 10 or 100. **Teaching point 4**: If the multiplier is less than one, the product is less than the multiplicand; if the multiplier is greater than one, the product is greater than the multiplicand. **Teaching point 5**: To divide any decimal fraction with up to two decimal places by a single-digit number, convert the decimal fraction to an integer by multiplying by 10 or 100, perform the resulting calculation using an appropriate strategy, then adjust the quotient by dividing by 10 or 100.
		Teaching point 1: Volume is the amount of space that something occupies.

2.20 **Multiplication with three factors and volume**	Use multiplication to calculate the volume of cuboids and shapes comprised of several cuboids; use division to solve associated inverse problems. Use associativity and commutativity to solve abstract multiplication problems with three factors.	**Teaching point 2**: Volume is measured in cubic units, such as cubic centimetres (cm^3) and cubic metres (m^3). **Teaching point 3**: The volume of a cuboid can be calculated by multiplying the length, width and height. **Teaching point 4**: Both the commutative law and the associative law can be applied when multiplying three or more numbers. **Teaching point 5**: The choice of which order to multiply in can be made according to the simplest calculation.
2.21 **Factors, multiples prime numbers and composite numbers**	Identify properties of factors and multiples including square and prime numbers, composite numbers, common and prime factors, and common multiples. Use factor pairs to solve problems efficiently.	**Teaching point 1**: Factors are positive integers that can be multiplied together to equal a given number. **Teaching point 2**: Systematic methods can be used to find all factors of a number; factors come in pairs; all positive integers have an even number of factors apart from square numbers, which have an odd number of factors; numbers with more than two factors are called composite numbers. **Teaching point 3**: Prime numbers are positive integers that have exactly two factors. **Teaching point 4**: A common factor is a factor that is shared by two or more numbers. A prime factor is a factor that is also a prime number. **Teaching point 5**: A multiple of a number is the product of that number and an integer; a common multiple is a multiple that is shared by two or more numbers. **Teaching point 6**: The factor pairs of '100' can be used to support efficient calculation.
2.22 **Combining multiplication with addition and subtraction**	Learn to combine multiplication with addition or subtraction. Learn to use brackets to change the order of operations. Build on knowledge of the distributive law.	**Teaching point 1**: Multiplication can be combined with addition and subtraction; when there are no brackets, multiplication is completed before addition or subtraction; when there are brackets, the calculation within the brackets is completed first. **Teaching point 2**: When adding or subtracting multiplication expressions that have a common factor, the distributive law can be applied.
2.23 **Multiplication strategies for larger numbers and long multiplication**	Develop strategies for multiplying two numbers with two or more digits, including adjusting strategies when multiplying by a power of ten, partitioning followed by multiplication and addition of partial products, and long multiplication.	**Teaching point 1**: When multiplying two numbers that are multiples of 10, 100 or 1,000, multiply the number of tens, hundreds or thousands and then adjust the product using place value. **Teaching point 2**: When multiplying two numbers where one number is a multiple of 10, 100 or 1,000, use short multiplication and adjust the product using place value. **Teaching point 3**: Two two-digit numbers can be multiplied by partitioning one of the factors, calculating partial products and then adding these partial products. This method can be extended to multiplication of three-digit numbers by two-digit numbers. **Teaching point 4**: *'Long multiplication'* is an algorithm involving multiplication, then addition of partial products, which supports multiplication of two numbers with two or more digits. **Teaching point 5**: Multiplication where one of the factors is a composite number can be carried out by multiplying one factor and then the other factor.
2.24 **Division: dividing by two-digit divisors**	Learn to divide by two-digit divisors, recording calculations using either the short or long division algorithm. Represent remainders in an appropriate way, according to the context, including using the short or long division algorithm to express remainders as decimal fractions.	**Teaching point 1**: Any two- or three-digit dividend can be divided by a two-digit divisor by skip counting in multiples of the divisor (quotient < 10); these calculations can be recorded using the short or long division algorithms. **Teaching point 2**: Any three- or four-digit dividend can be divided by a two-digit divisor using the short or long division algorithms (including quotient ≥ 10). **Teaching point 3**: When there is a remainder, the result can be expressed as a whole-number quotient and a whole-number remainder, as a whole-number quotient and a proper-fraction remainder, or as a decimal-fraction quotient.

2.25 **Using compensation to calculate**	Learn how multiplication and division calculations are affected when one element of the calculation is multiplied or divided by a scale factor.	**Teaching point 1**: For multiplication, if there is a multiplicative change to one factor, the product changes by the same scale factor. **Teaching point 2**: For division, if there is a multiplicative change to the dividend and the divisor remains the same, the quotient changes by the same scale factor. **Teaching point 3**: For division, if there is a multiplicative increase to the divisor and the dividend remains the same, the quotient decreases by the same scale factor; if there is a multiplicative decrease to the divisor and the dividend remains the same, the quotient increases by the same scale factor.
2.26 **Mean average and equal shares**	Understand the concept of mean average and learn how to find the mean of a set of data. Use the mean to compare sets of data and learn when it is appropriate to use the mean.	**Teaching point 1**: The mean is the size of each part when a quantity is shared equally. **Teaching point 2**: The mean is defined as the sum of all the numbers in a set of data divided by the number of numbers/values that make up the set of data. If we know the mean of a set of data and the number of numbers/values in that set, we can calculate the total of the set. The mean of a set changes if the total value of the set changes or if the number of numbers/values in the set changes. **Teaching point 3**: The mean can be used to compare data. **Teaching point 4**: The mean is not always an appropriate representation of a set of data.
2.27 **Scale factors, ratio and proportional reasoning**	Use bar modelling and ratio grids to reason about multiplicative relationships between two or more cardinal quantities, and explore correspondence problems. Extend understanding of scaling measures to make and interpret maps and scale/compare the dimensions of similar shapes.	**Teaching point 1**: Multiplication and division can be used to calculate unknown values in correspondence (cardinal comparison) problems. **Teaching point 2**: Multiplication and understanding of correspondence can be used to calculate the number of possible combinations of items. **Teaching point 3**: Scaling can be used to make and interpret maps. **Teaching point 4**: There is a proportional relationship between the dimensions of similar shapes; if the scale factor and the dimensions of one of the shapes is known, the dimensions of the similar shape can be calculated; if the dimensions of both of the shapes are known, the scale factor can be calculated.
2.28 **Combining division with addition and subtraction**	Learn to combine division with addition or subtraction. Revisit the use of brackets to change the order of operations. Build on knowledge of the distributive law.	**Teaching point 1**: Division can be combined with addition and subtraction; when there are no brackets, division is completed before addition or subtraction; when there are brackets, the calculation within the brackets is completed first. **Teaching point 2**: When adding or subtracting division expressions that have a common divisor, the distributive law can be applied.
2.29 **Decimal place-value knowledge, multiplication and division**	Develop efficient calculation strategies, and connect knowledge of multiplying and dividing by 10/100/1,000 to understanding of place value, including application to conversion between metric units of measure.	**Teaching point 1**: To multiply a number by 10/100/1,000, move the digits one/two/three places to the left; to divide a number by 10/100/1,000, move the digits one/two/three places to the right. **Teaching point 2**: Measures can be converted from one unit to another using knowledge of multiplication and division by 10/100/1,000.
2.30 **Multiplicative contexts: area and perimeter 2**	Build on earlier knowledge of area and perimeter. Learn to find the area of parallelograms and triangles by identifying the perpendicular height. Compare areas and perimeters and apply	**Teaching point 1**: The area of a parallelogram can be calculated by multiplying the base by the perpendicular height; all parallelograms with the same base and perpendicular height will have the same area. **Teaching point 2**: The area of a triangle can be calculated by multiplying the base by the perpendicular height and then dividing by two. **Teaching point 3**: Shapes with the same area can have different perimeters; shapes with the same perimeter can have different areas.

	scale factors to side-length, perimeter and area.	**Teaching point 4**: When a shape has been transformed by a scale factor, the perimeter is also transformed by the same scale factor.

Spine 3: Fractions

Spine number	Overview	Teaching points
3.0 **KS1 Guidance**	Cover the Key Stage 1 statutory requirements for fractions, including recognising, finding, naming and writing one-quarter, one-third, one-half/two-quarters, and three-quarters of an object, shape or quantity.	**1**: Name the fractions ‘one-half’, ‘one-quarter’ and ‘one-third’ in relation to a fraction of a length, shape or set of objects. **2**: Read and write the fraction notation $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, and relate this to a fraction of a length, shape or set of objects. **3**: Find half of numbers. **4**: Find $\frac{1}{3}$ or $\frac{1}{4}$ of a number. **5**: Find $\frac{2}{4}$ and $\frac{3}{4}$ of an object, shape, set of objects, length or quantity; recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$.
3.1 **Preparing for fractions: the part-whole relationship**	Identify parts and wholes of areas, lengths and sets. Identify equal and unequal parts; make judgements about the relative size of a part to a whole. Find the whole when the size of a part and number of equal parts is known.	**Teaching point 1**: Any element of a whole is a part; if a whole is defined, then a part of this whole can be defined. **Teaching point 2**: A whole can be divided into equal parts or unequal parts. **Teaching point 3**: The relative size of parts can be compared. **Teaching point 4**: If one of the equal parts and the number of equal parts are known, these can be used to construct the whole.
3.2	Learn to name and write unit fractions. Recognise and show unit fractions of areas, lengths and quantities. Relate numerators and denominators to parts and wholes; explore how the greater the	**Teaching point 1**: A whole can be divided into any number of equal parts.

Unit fractions: identifying, representing and comparing	denominators, the smaller the unit fraction.	**Teaching point 2**: Fraction notation can be used to describe an equal part of the whole. One equal part of a whole is called a unit fraction. Each unit fraction has a name. **Teaching point 3**: Fractional notation can be applied to represent one part of a whole in different contexts. **Teaching point 4**: Equal parts do not need to look the same. **Teaching point 5**: Unit fractions can be compared and ordered by looking at the denominator. The greater the denominator, the smaller the fraction. **Teaching point 6**: If the size of a unit fraction is known, the size of the whole can be worked out by repeated addition of that unit fraction.
3.3 **Non-unit fractions: identifying, representing and comparing**	Learn to name and write non-unit fractions, recognising them as multiples of unit fractions. Learn that fractions are numbers that can be positioned on a number line. Compare and order fractions with the same denominator or same numerator.	**<u>Non-unit fractions</u>** **Teaching point 1:** All non-unit fractions are made up of more than one of the same unit fraction. **Teaching point 2:** Non-unit fractions are written using the same convention as unit fractions. A non-unit fraction has a numerator greater than one. **Teaching point 3:** When the numerator and the denominator in a fraction are the same, the fraction is equivalent to one whole. **<u>Fractions as numbers</u>** **Teaching point 4:** All unit and non-unit fractions are numbers that can be placed on a number line. **Teaching point 5:** Repeated addition of a unit fraction results in a non-unit fraction. **Teaching point 6:** When the numerator and the denominator are the same, the value of the fraction is one. **<u>Comparing fractions</u>** **Teaching point 7:** Non-unit fractions with the same denominator can be compared. If the denominators are the same, then the greater the numerator, the greater the fraction. **Teaching point 8:** Non-unit fractions with the same numerator can be compared. If the numerators are the same, then the greater the denominator, the smaller the fraction.
3.4 **Adding and subtracting within one whole**	Explore how to add and subtract fractions within one whole where the denominators are the same. Apply prior knowledge of the inverse relationship of addition and subtraction with whole numbers, to fractions.	**Teaching point 1**: When adding fractions with the same denominators, just add the numerators. **Teaching point 2**: When subtracting fractions with the same denominators, just subtract the numerators. **Teaching point 3**: Addition and subtraction of fractions are the inverse of each other, just as they are for whole numbers. **Teaching point 4**: To subtract from one whole, first convert the whole to a fraction where the denominator and numerator are the same.
3.5	Meet mixed numbers and improper fractions, and learn to convert between them; compare, order and place them on a number line. Extend addition and	**Teaching point 1**: Quantities made up of both wholes and parts can be expressed as mixed numbers. **Teaching point 2**: Mixed numbers can be placed on a number line.

Working across one whole: improper fractions and mixed numbers	subtraction from within a whole to numbers greater than one whole.	**Teaching point 3**: Understanding how to compare and order proper fractions supports the comparison and ordering of mixed numbers. **Teaching point 4**: Mixed numbers can be partitioned and combined in the same way as whole numbers. **Teaching point 5**: Mixed numbers can be written as improper fractions. **Teaching point 6**: Improper fractions can be added and subtracted in the same way as proper fractions.
3.6 **Multiplying whole numbers and fractions**	Consider multiplication of whole numbers and proper fractions as both repeated addition and scaling. Understand that multiplication of a whole number by a proper fraction results in a smaller number.	**Teaching point 1**: Repeated addition of proper and improper fractions can be expressed as multiplication of a fraction by a whole number. **Teaching point 2**: Repeated addition of a mixed number can be expressed as multiplication of a mixed number by a whole number. **Teaching point 3**: Finding a unit fraction of a quantity can be expressed as a multiplication of a whole number by a fraction. **Teaching point 4**: A non-unit fraction of a quantity can be calculated by first finding a unit fraction of that quantity. **Teaching point 5**: If the size of a non-unit fraction is known, the size of the unit fraction and then the size of the whole can be found.
3.7 **Finding equivalent fractions and simplifying fractions**	Discover how equivalent fractions have the same proportional relationship between the numerator and denominator, and therefore have the same numerical value. Convert between equivalent fractions and simplify fractions.	**Teaching point 1**: When two fractions have different numerators and denominators to one another but share the same numerical value, they are called 'equivalent fractions'. **Teaching point 2**: Equivalent fractions share the same proportional (multiplicative) relationship between the numerator and denominator. Equivalent fractions can be generated by maintaining that relationship through the process of multiplication and division. **Teaching point 3**: Fractions can be simplified by dividing both the numerator and denominator by a common factor.
3.8 **Common denomination: more adding and subtracting**	Learn to add and subtract fractions with different denominators by first finding a common denominator. Compare fractions using a range of methods, including converting to a common denominator.	**Teaching point 1**: In order to add related fractions, first convert one fraction so that both share the same denominator (a *'common denominator'*). **Teaching point 2**: To subtract related fractions, first convert one fraction so that both share a common denominator. **Teaching point 3**: The common denominator method can be extended to adding and subtracting non unit related fractions. **Teaching point 4**: To add and subtract *non-related* fractions, the product of the two denominators provides a common denominator. **Teaching point 5**: Converting to common denominators is one of several methods that can be used to compare fractions.
3.9 **Multiplying and dividing fractions by a whole number**	Explore how to multiply two fractions. Learn how to divide a fraction by a whole number by first converting to an equivalent multiplication. Review how multiplying by a proper fraction makes a number smaller.	**Teaching point 1**: When a fraction is multiplied by a proper fraction, it makes it smaller. To multiply two fractions, multiply the numerators and multiply the denominators. **Teaching point 2**: When a fraction is divided by a whole number, it makes it smaller. To divide a fraction by a whole number, convert it to an equivalent multiplication. **Teaching point 3**: A more efficient method can be used to divide a fraction by a whole number when the whole number is a factor of the numerator.

3.10 **Linking fractions, decimals and percentages**	Make connections between fractions and previous work on decimals. Learn common fraction and decimal equivalences. Understand that percentages tell us about the proportion being considered. Find percentages of quantities	**Teaching point 1**: Some fractions are easily converted to decimals. **Teaching point 2**: These fraction–decimal equivalents can be found throughout the number system. **Teaching point 3**: Fraction–decimal equivalence can sometimes be used to simplify calculations. **Teaching point 4**: *'Percent'* means number of parts per hundred. A percentage can be an operator on a quantity, indicating the proportion of a quantity being considered. **Teaching point 5**: Percentages have fraction and decimal equivalents. **Teaching point 6**: If the value of a whole is known, a percentage of that number or amount can be calculated.

The Shanghai Pedagogy

The Shanghai method of teaching is a whole-class teaching method that builds thorough understanding develops higher-order thinking and is supported by the use of high-quality textbooks.

The Shanghai pedagogy is based on:

Typical Lesson Structure