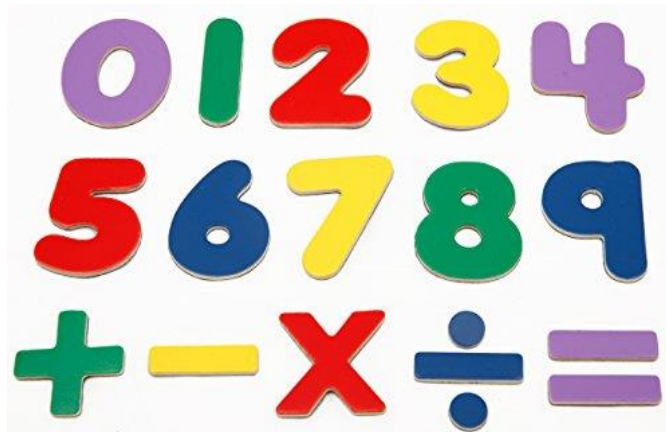


Mental Calculation Strategies for Y1-Y6



Diane Andrews, Maths Consultant

(With thanks to Judith Lambert,
Ivydale Primary School)

October 2017

INTRODUCTION

This **mental calculation strategies** policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014).

It provides guidance on effective mental strategies for calculation and gives year-by-year expectations of a range of calculations that children should be able to do mentally (including with jottings/informal recording).

The content is set out under the following headings: addition & subtraction strategies; multiplication and division strategies.

This guidance has been aligned with the Southwark Medium Term Plans (revised 2016).

Our aim is that children will use mental methods (including with the use of jottings/informal recording) as their first port of call, when appropriate.

However, for calculations that they cannot do mentally, they will need to use an efficient written method accurately and with confidence (see **written calculation policy** – updated April 2017).

Underpinning skills and knowledge needed to calculate mentally

- The ability to count in a variety of ways, both forwards and backwards
- A secure sense of the number system
- An understanding of place value
- Recall of number bonds
- Recall of multiplication and division facts
- An understanding of mathematical vocabulary and signs associated with calculation

Principles of teaching mental calculation

- Ensure the underpinning skills and knowledge are secure
- Commit regular time to teaching mental calculation strategies
- Select and use appropriate resources, models and images
- Encourage the use of jottings/ informal recording
- Teach a range of mental strategies
- Develop quick and efficient strategies, choosing the most appropriate method for the calculation
- Give children the opportunity to explain, share and reason about methods

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With thanks to Judith Lambert, Ivydale Primary School, for her contributions

Underpinning skills and knowledge: end of year expectations

Y1	<ul style="list-style-type: none"> • Count to and across 100, forwards and backwards, in ones, beginning with 0 or 1 or from any given number • Given a number, identify one more/one less • Recognise place value in teen numbers using practical apparatus and begin to recognise place value in other two-digit numbers • Read, write and interpret mathematical statements involving addition and subtraction, including the signs +, -, =, and understand the associated vocabulary • Recognise the relationship between addition and subtraction • Recall addition/subtraction facts to 10 and within 10 • Derive number bonds to 20 and within 20 • Count in multiples of twos, fives and tens (to the 10th multiple) • Understand and use the vocabulary (but not the signs) associated with multiplication and division in practical contexts • Recall doubles up to double ten (10 + 10) and find the corresponding halves
Y2	<ul style="list-style-type: none"> • Count to at least 100 in ones and in tens from 0 or any number, forwards and backwards • Given a number, identify 10 more/10 less • Recognise the place value of each digit in a two-digit number • Use the vocabulary associated with addition and subtraction • Recall addition/subtraction facts to 20 • Derive addition/subtraction facts of multiples of 10 to 100 e.g. 60 + 40 = 100 • Know that addition of two numbers can be done in any order (commutative) but that subtraction of one number from another cannot • Recognise and use the inverse relationship between addition and subtraction • Use estimation to check that an answer to a calculation is reasonable • Count in multiples of 2, 3, 5 from 0, forwards and backwards (to the 12th multiple) • Recall multiplication/division facts for the 2, 5 and 10 times table to the 12th multiple • Read, write and interpret mathematical statements involving multiplication and division, including the signs x, ÷ and =, and understand and use the associated vocabulary • Know that multiplication of two numbers can be done in any order (commutative) but that division of one number by another cannot • Recall the doubles of multiples of 10 to 100 (e.g. double 40 is 80) and recall the related halves (e.g. half of 80 is 40)
Y3	<ul style="list-style-type: none"> • Given a number, identify 10 or 100 more/less • Recognise the place value of each digit in a three-digit number • Recall addition and subtraction facts for multiples of 10 to 100 • Derive addition and subtraction facts for multiples of five to 100 • Derive addition and subtraction facts for multiples of 100 to 1000 • Recognise the inverse relationship between addition and subtraction • Estimate the answer to a calculation and use inverse operations to check • Count in multiples of 2, 3, 4, 5, 8, 10, 50 and 100 from 0, forwards and backwards (to the 12th multiple) • Recall multiplication/division facts for the 2, 3, 4, 5, 8 and 10 times tables • Understand the effect of multiplying/dividing numbers by 10 • Understand the commutative properties of addition and of multiplication • Recognise and use the inverse relationship between multiplication and division • Derive doubles of all two-digit numbers (e.g. double 42 is 84) and the corresponding halves (half of 84 is 42)

<p>Y4</p>	<ul style="list-style-type: none"> • Given a number, identify 10, 100 or 1000 more/less • Recognise the value of each digit in a four-digit number • Round any number to the nearest 10, 100 or 1,000 • Recognise the place value of each digit in a decimal number with up to two decimal places • Round decimal numbers with one decimal places to the nearest whole number • Find pairs of decimal numbers that total one (e.g.0.4 and 0.6) • Derive addition and subtraction facts for pairs of numbers that total 100 • Know addition/subtraction facts for multiples of 100 that total 1,000 • Derive addition and subtraction facts for multiples of 50 to 1,000 and multiples of 10 to 1,000 • Count in multiples of 2, 3, 4, 5, 6, 7,8, 9,10, 11, 12, 25, 50, 100 and 1000 from 0, forwards and backwards (to the 12th multiple) • Recall multiplication and division facts for multiplication tables up to 12 x 12 • Understand the effect of multiplying by 0 or 1 and dividing by 1 • Recognise and identify factor pairs • Understand the effect of multiplying/dividing numbers by 10/100, including decimal numbers • Recall doubles of two-digit numbers and derive doubles of three-digit numbers and find the corresponding halves • Estimate the answer to a calculation, including the use of rounding, and use inverse operations to check
<p>Y5</p>	<ul style="list-style-type: none"> • Given a number identify 10/ 100/ 1,000/ 10,000 more or less • Recognise the place value of each digit in a six-digit whole number • Round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000 • Recognise the place value of each digit in a decimal number with up to three decimal places • Round decimal numbers with two decimal places to the nearest whole number or to one decimal place • Derive complements of 1 e.g. 0.83 and 0.17 = 1 • Count in multiples of 3, 4, 6, 7, 8, 9, 11,12, 25, 50,100 and 1,000 forwards and backwards • Consolidate multiplication and division facts for multiplication tables up to 12 x 12 • Find all factor pairs of a given number; find all common factors for a pair of numbers; identify multiples • Derive all square numbers to 12² (12 x 12 = 144) • Understand the effect of multiplying/dividing whole numbers, and decimal numbers with up to two decimal places, by 10, by 100 and by 1,000 • Derive doubles of three-digit and four-digit numbers (and decimal numbers with up to two decimal places) and find the corresponding halves • Estimate the answer to a calculation and use inverse operations to check
<p>Y6</p>	<ul style="list-style-type: none"> • Consolidate all end of year expectations for Y5 and... • Recognise the place value of each digit in a seven-digit whole number • Recall multiplication/division facts for all multiplication tables up to 12 x 12 with fluency • Identify factors, common factors, common multiples and prime factors • Recall all square numbers to 12² (12 x 12 = 144) • Understand the effect of multiplying/dividing whole numbers, and decimal numbers with up to three decimal places, by 10, by 100 and by 1,000 • Understand the order of operations using brackets (BODMAS)

Progression in addition and subtraction strategies

Year 1

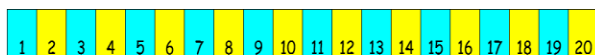
Underpinning skills (end of year expectation)

- Count to and across 100, forwards and backwards, in ones, beginning with 0 or 1 or from any given number
- Given a number, identify one more/one less
- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and the equals (=) sign and understand and use the associated vocabulary
- Recognise the relationship between addition and subtraction
- Recall addition/subtraction facts to 10 and within 10
- Derive addition/subtraction facts to 20 and within 20
- Recall doubles up to double ten (10 + 10) and find the corresponding halves
- Recognise place value in teen numbers using practical apparatus and begin to recognise place value in other two-digit numbers

Strategies

Counting on and back in ones

Children will count on or back in ones, from 0, 1 or any number, including with the use of a marked number line and/ or a number track:



- 8 + 6 count on in ones from 8
- 16 + 3 count on in ones from 16
- 18 + 4 count on in ones from 18
- 10 - 4 count back in ones from 10
- 12 - 5 count back in ones from 12
- 17 - 4 count back in ones from 17
- 20 - 8 count back in ones from 20



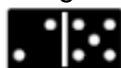
Use a counting stick to count forwards and backwards in ones, from any number, within 100
Ask children to count from 0, 1 or any number, in ones. When you clap, they count backwards. On the next clap, they count forwards, and so on...

Use counting songs and rhymes

Re-ordering numbers when adding

Children will know that it can sometimes be easier to re-order numbers when adding to start with the largest number, understanding that addition can be done in any order:

$$2 + 5 = 7$$



$$5 + 2 = 7$$



3 + 12 becomes 12 + 3

6 + 18 becomes 18 + 6

Partitioning numbers in different ways

Children will begin to use their knowledge of place value to add or subtract, without using counting strategies:

$$10 + 4 = 14$$

$$16 - 6 = 10$$

$$20 + 3 = 23$$

Children will begin to use their knowledge of number bonds to 10 (then to 20) to partition when adding and subtracting:

$$7 + 4 = 7 + 3 + 1$$

$$18 + 3 = 18 + 2 + 1$$

$$14 - 6 = 14 - 4 - 2$$

Adding using recall of doubles

Children will use their knowledge of doubles to add:

$$5 + 5 \text{ is double } 5$$



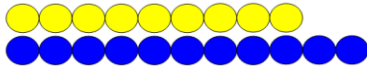
Children will begin to use their knowledge of doubles to add near doubles:

$$5 + 6 \text{ is double } 5 \text{ add } 1$$

Finding the difference by counting on

Children will use complementary addition to count on from the smaller number to the larger number to find a small difference, including with the use of resources, such as counters or a number track:

11 – 9 count up from 9 to 11 to find the difference



What's the difference between nine and eleven?

18 – 15 count up from 15 to find the difference

21 – 18 count up from 18 to 21 to find the difference

Rapid recall

Give children the opportunity to respond rapidly to oral and written questions, using a range of vocabulary:

$$7 + 3$$

Eight plus four

$$18 \text{ add } 2$$

One more than 19

Twelve take away four

$$15 \text{ minus } 6$$

$$20 - 5$$

One less than 21

The difference between ten and fourteen

Using related calculations

Children will use knowledge of place value and related calculations:

$$17 + 3 = 20 \text{ using } 7 + 3 = 10$$

Children will use their understanding of the relationship between addition and subtraction and that addition can be done in any order, using resources to support understanding:

$$7 + 3 = 10$$

$$3 + 7 = 10$$

$$10 - 3 = 7$$

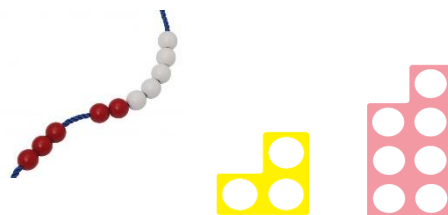
$$10 - 7 = 3$$

$$17 + 3 = 20$$

$$3 + 17 = 20$$

$$20 - 3 = 17$$

$$20 - 17 = 3$$



Year 2

Underpinning skills (end of year expectation)

- Count to at least 100 in ones and in tens from 0 or any number, forwards and backwards
- Given a number, identify 10 more/10 less
- Recognise the place value of each digit in a two-digit number
- Use the vocabulary associated with addition and subtraction
- Recall addition/subtraction facts to 20
- Derive addition/subtraction facts of multiples of ten to 100 e.g. $60 + 40 = 100$
- Know that addition of two numbers can be done in any order (commutative) but that subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction
- Recall the doubles of multiples of 10 to 100 (e.g. double 40 is 80) and recall the related halves (e.g. half of 80 is 40)
- Use estimation to check that an answer to a calculation is reasonable

Strategies

Counting on and back in tens and ones

Children will use their understanding of place value to support counting on or back, including with the use of a 100 square/ 200 grid to support and/or a number line:

$42 + 5$ count on in ones from 42

$42 + 10$ count on ten from 42

$42 + 30$ count on in tens from 42

$42 + 35$ count on in tens then ones from 42

$56 - 4$ count back in ones from 56

$56 - 10$ count back ten from 56

$56 - 20$ count back in tens from 56

$56 - 24$ count back in tens then ones from 56

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Use a counting stick to count forwards and backwards in ones from any number and to count forwards and backwards in tens from any number, to at least 100

Ask children to count from any two-digit number in tens. When you clap, they count on in ones. On the next clap, they count on in tens, and so on...

Partitioning numbers into tens and ones

Children will use their understanding of place value to partition numbers into tens and ones:



$$30 + 2 = 32$$

$$32 - 2 = 30$$

Children will partition both numbers into tens and ones and then re-order and add

$$25 + 14 = 20 + 5 + 10 + 4 = 20 + 10 + 5 + 4$$

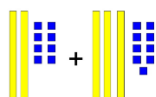
$$45 + 24 = 40 + 20 + 5 + 4 = 40 + 20 + 5 + 4$$

Consider the use of base ten resources to support

$$15 + 13 = 10 + 5 + 10 + 3 = 10 + 10 + 5 + 3$$



$$28 + 39 = 20 + 8 + 30 + 9 = 20 + 30 + 8 + 9$$

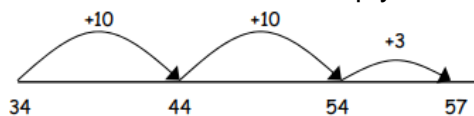


Or, children will keep the first number as it is and partition the second number

$$25 + 14 = 25 + 10 + 4$$

$$34 + 23 = 34 + 20 + 3 = 34 + 10 + 10 + 3$$

Consider the use of an empty number line to record jottings



Children will partition the second number to subtract

$$68 - 24 = 68 - 20 - 4$$

$$56 - 34 = 56 - 30 - 4$$

Consider the use of base ten resources or an empty number line to count back

Children will use their knowledge of number bonds and place value to partition when adding and subtracting, bridging through multiples of ten, including with the use of empty number lines:

$$27 + 4 = 27 + 3 + 1$$

$$34 - 6 = 34 - 4 - 2$$

Re-ordering numbers when adding

Children will know that it can sometimes be easier to re-order numbers when adding:

Re-order to start with the largest number and understand the commutative property of addition:

$$23 + 56 \text{ becomes } 56 + 23$$

Re-order to find pairs that total 10 (or 20) when adding three small numbers:

$$8 + 9 + 2 \text{ becomes } 8 + 2 + 9 = 10 + 9$$

$$16 + 2 + 4 \text{ becomes } 16 + 4 + 2 = 20 + 2$$

Add and subtract multiples of 10 and adjust

Children will use their knowledge of adding and subtracting 10 to add/subtract 9 or 11, including with the use of a 100 square or an empty number line:

$$42 + 9 = 42 + 10 - 1$$

$$42 + 11 = 42 + 10 + 1$$

$$42 - 9 = 42 - 10 + 1$$

$$42 - 11 = 42 - 10 - 1$$

Adding near doubles

Children will use their knowledge of doubles to add near doubles:

$$6 + 7 \text{ is double } 6 \text{ and add } 1$$

$$10 + 11 \text{ is double } 10 \text{ add } 1$$

$$12 + 13 \text{ is double } 12 \text{ and add } 1$$

$$20 + 19 \text{ is double } 20 \text{ and subtract } 1$$

$$40 + 39 \text{ is double } 40 \text{ and subtract } 1$$

Finding the difference by counting on

Children will use complementary addition to count on from the smaller number to the larger number to find a small difference, including with the use of an empty number line:



15 - 8 count on from 8 to 15 to find the difference

42 - 38 count on from 38 to 42 to find the difference

92 - 78 count on from 78 to 92 to find the difference

Rapid recall

Give children the opportunity to respond rapidly to oral and written questions, using a range of vocabulary:

$27 + 3$

Ten more than 34

56 plus 12

The total of 50 and 4

60 add 40

$20 - 6$

100 subtract 50

Ten less than 86

65 minus 5

The difference between 29 and 31

Using related calculations

Children will use their understanding of place value and related calculations:

$37 + 3 = 40$ using $7 + 3 = 10$

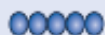
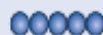


$50 + 40 = 90$ using $5 + 4 = 9$

$100 - 30 = 70$ using $10 - 3 = 7$

Children will use their knowledge that addition can be done in any order (addition is commutative):

$70 + 30 =$	$30 + 70 =$
?	?
70 30	30 70

Children will use their understanding of inverse operations and the commutative property of addition:

		$20 = 12 + 8$	$8 + 12 = 20$
		$20 - 8 = 12$	$20 - 12 = 8$

$60 + 40 = 100$ therefore...

$40 + 60 = 100$

$100 - 40 = 60$

$100 - 60 = 40$

Children will use inverse operations to solve empty box questions

$48 + \square = 54$

$95 - \square = 88$

Year 3

Underpinning skills (end of year expectation)

- Given a number, identify 10 or 100 more/less
- Recognise the place value of each digit in a three-digit number
- Recall addition and subtraction facts for multiples of 10 to 100
- Derive addition and subtraction facts for multiples of five to 100
- Derive addition and subtraction facts for multiples of 100 to 1000
- Understand the commutative properties of addition and the inverse relationship between addition and subtraction
- Derive doubles of all two-digit numbers (e.g. double 42 is 84)
- Estimate the answer to a calculation and use inverse operations to check

Strategies

Counting on and back in hundreds, tens and ones

Children will use their understanding of place value to support counting on or back, including with The use of a 200 grid and/or an empty number line:

- 82 + 30 count on in tens from 82
- 142 + 32 count on in tens and ones from 142
- 82 + 100 count on one hundred from 82
- 142 + 100 count on one hundred from 142
- 462 + 300 count on in hundreds from 462
- 136 – 40 count back in tens from 136
- 156 – 25 count back in tens and ones from 156
- 452 – 100 count back one hundred from 452
- 752 – 400 count back in hundreds from 752

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

Use a counting stick to count on or back in tens or hundreds from any number within 1,000

Ask children to count from any two-digit or three-digit number in tens. When you clap, they count backwards. On the next clap, they count forwards, and so on...

Partitioning numbers in different ways

Children will use their understanding of place value to partition numbers:

Children partition both numbers into tens and ones and then re-order and add
 $63 + 54 = 60 + 3 + 50 + 4 = 60 + 50 + 3 + 4 = 110 + 7$

Consider the use of base ten resources to support

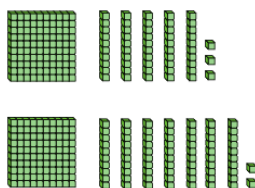
$$48 + 36 = 40 + 8 + 30 + 6 = 40 + 30 + 8 + 6$$



Children partition both numbers into hundreds, tens and ones and then re-order and add
 $123 + 235 = 100 + 20 + 3 + 200 + 30 + 5 = 100 + 200 + 20 + 30 + 3 + 5 = 300 + 50 + 8$

Consider the use of base ten resources to support

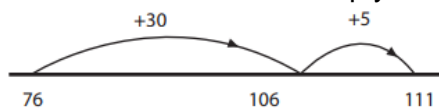
$$154 + 172$$



Or, children will keep the first number as it is and partition the second number

$$76 + 35 = 76 + 30 + 5$$

Consider the use of an empty number line to record jottings



Encouraging children to use number lines in this way provides a mental image that can assist with mental calculations

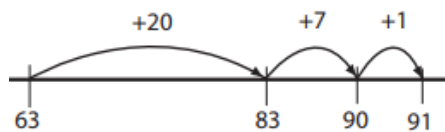
$$125 + 34 = 125 + 30 + 4$$

$$146 + 135 = 196 + 100 + 30 + 5$$

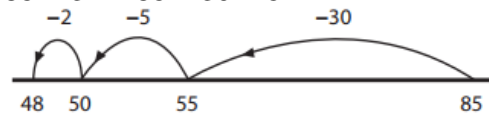
$$236 - 142 = 236 - 100 - 40 - 2$$

Children use their knowledge of number bonds and place value to partition when adding and subtracting, bridging through multiples of 10 or 100

$$63 + 28 = 63 + 20 + 7 + 1$$



$$85 - 37 = 85 - 30 - 5 - 2$$



$$127 + 8 = 127 + 3 + 5$$

$$234 - 5 = 234 - 4 - 1$$

$$103 - 7 = 103 - 3 - 4$$

Re-ordering numbers when adding

Children will know that it can sometimes be easier to re-order numbers when adding:

Re-order to start with the largest number and understand the commutative property of addition

$$23 + 356 \text{ becomes } 356 + 23$$

Re-order to find pairs that total multiples of 10 when adding/subtracting three small numbers

$$11 + 15 + 9 \text{ becomes } 11 + 9 + 15 = 20 + 15$$

$$92 + 12 + 8 \text{ becomes } 92 + 8 + 12 = 100 + 12$$

$$42 - 7 - 2 \text{ becomes } 42 - 2 - 7 = 40 - 7$$

Add and subtract multiples of 10 or 100 and adjust

Children use their knowledge of adding and subtracting multiples of 10 or 100 to add/subtract 9, 19, 29 or 11, 21, 31 or 99, 101... including with the use of an empty number line:

$$142 + 19 = 142 + 20 - 1$$

$$342 + 21 = 342 + 20 + 1$$

$$142 - 19 = 142 - 20 + 1$$

$$442 + 99 = 442 + 100 - 1$$

$$345 - 99 = 345 - 100 + 1$$

Adding near doubles

Children use their knowledge of doubles to add near doubles:

$$15 + 16 = \text{double } 15 \text{ add } 1$$

$$25 + 26 = \text{double } 25 \text{ and add } 1$$

$$50 + 60 = \text{double } 50 \text{ and add } 10$$

Finding the difference by counting on

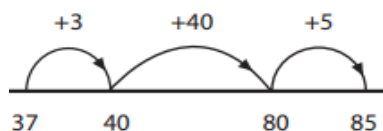
Children use complementary addition to count on from the smaller number to the larger number to find a small difference, including with the use of an empty number line:

104 – 95 count up from 95

202 – 198 count up from 198

212 – 199 count up from 199

It is sometimes easier to count on to find a difference even if the difference isn't small



85 – 37 count up from 37

'37 and 3 makes 40 and 40 makes 80 and 5 makes 85. So add 3 + 40 + 5 to get the answer'

Encouraging children to use number lines in this way provides a mental image that can assist with mental calculations

Rapid recall

Give children the opportunity to respond rapidly to oral and written questions using a range of vocabulary:

70 + 30

Ten more than 194

The total of 500 and 400

Add 100 to 245

The sum of 9, 10 and 11

Increase 85 by 40

100 – 20

1,000 subtract 400

100 less than 186

265 minus 60

Decrease 200 by 30

The difference between 99 and 101

Using related calculations

Children will use knowledge of place value and related calculations:

140 + 150 = 290 using 14 + 15 = 29

300 + 700 = 1,000 using 30 + 70 = 100

Children will continue to use the inverse relationship between addition and subtraction and the commutative property of addition:

145 + 36 = 181 therefore...

36 + 145 = 181

181 – 36 = 145

181 – 145 = 36

Children will use inverse operations to solve empty box questions

148 + = 154

195 – = 184

+ 100 = 345

If you know that 700 + 300 = 1,000, what else do you know?

Year 4

Underpinning skills (end of year expectation)

- Given a number, identify 10, 100 or 1000 more/less
- Recognise the place value of each digit in a four-digit number
- Round any number to the nearest 10, 100 or 1,000
- Recognise the place value of each digit in a decimal number with up to two decimal places
- Round decimal numbers with one decimal places to the nearest whole number
- Find pairs of decimal numbers that total one (e.g. $0.6 + 0.4$)
- Know addition/subtraction facts for multiples of 100 that total 1,000
- Derive addition and subtraction facts for all pairs of numbers that total 100 e.g. $68 + 32$
- Derive addition and subtraction facts for multiples of 50 to 1,000 and multiples of 10 to 1,000
- Recall doubles of two-digit numbers and derive doubles of three-digit numbers
- Estimate the answer to a calculation, including using the skill of rounding, and use inverse operations to check

Strategies

Counting on and back in thousands, hundreds, tens and ones

Children will use their understanding of place value to support counting on or back, including with the use of an empty number line:

$564 + 400$ count on in hundreds from 564

$960 + 200$ count on in hundreds from 960

$1,250 + 68$ count on in tens and then ones from 1,250

$4,458 + 1,000$ count on one thousand from 4,458

$4,450 + 3,000$ count on in thousands from 4,450

$936 - 40$ count back in tens from 936

$1,856 - 35$ count back in tens and ones from 1,856

$1,456 - 500$ count back in hundreds from 1,456

$6,452 - 1,000$ count back one thousand from 6,452

$8,450 - 5,000$ count back in thousands from 8,450

Ask children to count on in hundreds from any three-digit number. When you clap, they count on in tens. On the next clap, they count back in hundreds, and so on...

Partitioning numbers in different ways

Children will partition both numbers into hundreds, tens and ones and then add

$163 + 224 = 100 + 200 + 60 + 20 + 3 + 4 = 300 + 80 + 7$

Consider the use of base ten resources to support

Or, children will keep the first number as it is and partition the second number

$163 + 244 = 163 + 200 + 40 + 4$

$625 - 434 = 625 - 400 - 30 - 4$

$1,567 + 1,349 = 1,567 + 1000 + 300 + 40 + 9$

Consider the use of an empty number line to record jottings

Children will extend their understanding of place value to partition decimal numbers and then add

$5.0 + 3.5 = 5.0 + 3.0 + 0.5$

$4.6 + 2.3 = 4.0 + 2.0 + 0.6 + 0.3 = 6.0 + 0.9$

Children will use their knowledge of number bonds and place value to partition when adding and subtracting, bridging through multiples of 10, 100 or 1,000, including with the use of an empty number line to record jottings:

$127 + 83 = 127 + 3 + 80 = 130 + 80$

$234 - 15 = 234 - 14 - 1 = 220 - 1$

$488 + 15 = 488 + 12 + 3$

Re-ordering numbers when adding

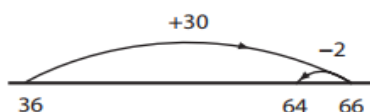
Children will know that it can sometimes be easier to re-order numbers when adding:
Re-order to start with the largest number and understand the commutative property of addition
 $210 + 856$ becomes $856 + 210$

Re-order to find pairs that total multiples of 1, 10 or 100 when adding/subtracting three small numbers
 $88 + 65 + 12$ becomes $88 + 12 + 65 = 100 + 65$
 $25 + 36 + 75$ becomes $75 + 25 + 36 = 100 + 36$
 $50 + 82 + 150$ becomes $150 + 50 + 82 = 200 + 82$
 $142 - 5 - 12$ becomes $142 - 12 - 5 = 130 - 5$
 $0.3 + 1.5 + 0.7$ becomes $1.5 + 0.7 + 0.3 = 1.5 + 1$

Add and subtract multiples of 10 or 100 and adjust

Children use their knowledge of adding and subtracting multiples of 10 or 100 and adjusting to add/subtract, including with the use of an empty number line:

$$36 + 28 = 36 + 30 - 2 = 64 \quad (28 \text{ rounds up to } 30)$$



Encouraging children to use number lines in this way provides a mental image that can assist with mental calculations

$$542 + 29 = 542 + 30 - 1 \quad (29 \text{ rounds up to } 30)$$
$$458 + 99 = 458 + 100 - 1 \quad (99 \text{ rounds up to } 100)$$
$$942 - 18 = 942 - 20 + 2 \quad (18 \text{ rounds up to } 20)$$
$$942 + 99 = 942 + 100 - 1 \quad (99 \text{ rounds up to } 100)$$
$$1,256 - 98 = 1,256 - 100 + 2 \quad (98 \text{ rounds up to } 100)$$
$$2,565 + 999 = 2,565 + 1,000 - 1 \quad (999 \text{ rounds up to } 1,000)$$

Adding near doubles

Children use their knowledge of doubles to add near doubles:

$$35 + 34 = \text{double } 35 \text{ and subtract } 1$$
$$45 + 46 = \text{double } 45 \text{ and add } 1$$
$$60 + 62 = \text{double } 60 \text{ and add } 2$$
$$150 + 152 = \text{double } 150 \text{ and add } 2$$

Finding the difference by counting on

Children will use complementary addition to count on from the smaller number to the larger number to find a **small** difference, including with the use of an empty number line:

$$504 - 498 \text{ count up from } 498$$
$$902 - 887 \text{ count up from } 887$$
$$1,004 - 998 \text{ count up from } 998$$

Rapid recall

Give children the opportunity to respond rapidly to oral and written questions using a range of vocabulary:

$$700 + 300$$
$$100 \text{ more than } 984$$
$$\text{The total of } 250 \text{ and } 150$$
$$\text{The sum of } 30 + 40 + 50$$
$$100 - 25$$
$$100 \text{ less than } 1,086$$
$$1,000 \text{ minus } 250$$
$$\text{Decrease } 1,000 \text{ by } 400$$
$$\text{The difference between } 198 \text{ and } 205$$

Using related calculations

Children will use knowledge of place value and related calculations:

Use $45 + 23 = 68$ to solve $450 + 230$ and $4.5 + 2.3$

Children will continue to use the inverse relationship between addition and subtraction and the commutative property of addition:

$850 + 150 = 1,000$ therefore...

$150 + 850 = 1,000$

$1,000 - 150 = 850$

$1,000 - 850 = 150$

If you know that $1,000 - 250 = 750$, what else do you know?

Children will use inverse operations to solve empty box questions

$$548 + \square = 654$$

$$995 - \square = 894$$

$$850 + \square = 1,000$$

Year 5

Underpinning skills (end of year expectation)

- Given a number identify 10/ 100/ 1,000/ 10,000 more or less
- Recognise the place value of each digit in a six-digit whole number
- Round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000
- Recognise the place value of each digit in a decimal number with up to three decimal places
- Round decimal numbers with two decimal places to the nearest whole number or to one decimal place
- Derive complements of 1 e.g. 0.83 and $0.17 = 1$
- Derive doubles of three-digit and four-digit numbers (and decimal numbers with up to two decimal places) and find the corresponding halves
- Estimate the answer to a calculation, including using the skill of rounding, and use inverse operations to check

Strategies

Counting on and back in tens of thousands, thousands, hundreds, tens and ones

Children will use their understanding of place value to support counting on or back, including with the use of an empty number line:

$864 + 500$ count on in hundreds from 864

$1,960 + 200$ count on in hundreds from 1,960

$1,250 + 268$ count on in hundreds, tens and then ones from 1,250

$9,458 + 3,000$ count on in thousands from 9,458

$25,250 + 3,500$ count on in thousands and then hundreds from 25,250

$456,000 + 40,000$ count on in tens of thousands from 456,000

$1,936 - 740$ count back in hundreds and then tens from 1,936

$5,856 - 235$ count back in hundreds, tens and ones from 5,856

$16,400 - 5,000$ count back in thousands from 16,400

$61,450 - 30,000$ count back in tens of thousands from 61,450

Ask children to count on in thousands from any three-digit number. When you clap, they count on in hundreds. On the next clap, they count back in thousands, and so on...

Partitioning numbers in different ways

Children partition the second number into thousands, hundreds, tens and ones and then add/subtract:

$$540 + 284 = 540 + 200 + 80 + 4$$

$$2,456 + 2,500 = 2,456 + 2,000 + 500$$

$$1,650 - 240 = 1,650 - 200 - 40$$

Consider the use of an empty number line to record jottings

Children use their understanding of place value to partition decimal numbers and then add/subtract

$$2.75 + 3.25 = 2.75 + 3.00 + 0.25 = 5.75 + 0.25$$

$$16.3 + 3.2 = 16.3 + 3.0 + 0.2$$



Children use their knowledge of number bonds and place value to partition when adding and subtracting, bridging through multiples of 10, 100 or 1,000

$$896 + 134 = 896 + 4 + 130 = 900 + 130$$

$$2,165 - 47 = 2,165 - 45 - 2$$

$$1,995 + 245 = 1,995 + 5 + 200 + 40$$

$$3.8 + 2.6 = 3.8 + 0.2 + 2.4 = 4.00 + 2.4$$

Re-ordering numbers when adding

Children will know that it can sometimes be easier to re-order numbers when adding:

Re-order to start with the largest number and understand the commutative property of addition

$$230 + 1,856 \text{ becomes } 1,856 + 230$$

Re-order to find pairs that total multiples of 1, 10, 100 or 1,000 when adding/subtracting three numbers

$$488 + 65 + 12 \text{ becomes } 488 + 12 + 65 = 500 + 65$$

$$750 + 73 + 250 \text{ becomes } 750 + 250 + 73 = 1,000 + 73$$

$$142 - 5 - 12 \text{ becomes } 142 - 12 - 5 = 130 - 5$$

$$158 + 47 - 38 \text{ becomes } 158 - 38 + 47 = 120 + 47$$

$$0.35 + 1.5 + 0.65 \text{ becomes } 0.65 + 0.35 + 1.5 = 1.0 + 1.5$$

$$1.7 + 2.8 + 0.3 \text{ becomes } 1.7 + 0.3 + 2.8 = 2 + 2.8$$

Add and subtract multiples of 10, 100 or 1,000 and adjust

Children will use their knowledge of adding and subtracting multiples of 10, 100 or 1,000 and adjusting to add/subtract, including with the use of an empty number line:

$$542 + 29 = 542 + 30 - 1 \text{ (29 rounds up to 30)}$$

$$942 - 38 = 942 - 40 + 2 \text{ (38 rounds up to 40)}$$

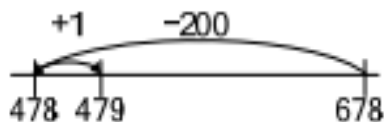
$$942 + 99 = 942 + 100 - 1 \text{ (99 rounds up to 100)}$$

$$1,856 - 201 = 1,856 - 200 - 1 \text{ (201 rounds down to 200)}$$

$$1,256 - 98 = 1,256 - 100 + 2 \text{ (98 rounds up to 100)}$$

$$2,565 + 999 = 2,565 + 1,000 - 1 \text{ (999 rounds up to 1,000)}$$

$$678 - 199 = 678 - 200 + 1 \text{ (199 rounds up to 200)}$$



Encouraging children to use number lines in this way provides a mental image that can assist with mental calculations

Adding near doubles

Children will use their knowledge of doubles to add near doubles:

$$1.5 + 1.6 = \text{double } 1.5 \text{ and add } 0.1$$

$$125 + 126 = \text{double } 125 \text{ and add } 1$$

$$500 + 600 = \text{double } 500 \text{ and add } 100$$

$$390 + 380 = \text{double } 400 \text{ and subtract } 10 \text{ and then subtract } 20$$

Finding the difference by counting on

Children will use complementary addition to count on from the smaller number to the larger number to find a **small** difference, including with the use of an empty number line:

$$904 - 898 \text{ count up from } 898$$

$$1,010 - 998 \text{ count up from } 998$$

$$1,002 - 877 \text{ count up from } 877$$

$$2,017 - 1,998 \text{ count up from } 1,998$$

$$8,004 - 6,999 \text{ count up from } 6,999$$

Rapid recall

Give children the opportunity to respond rapidly to oral and written questions, using a range of vocabulary:

The total of 250 and 150

$7,000 + 3,000$

The sum of 2,500 and 2,500

The total of $300 + 500 + 700$

100 more than 950

Increase 850 by 300

$1,000 - 150$

100 less than 1,086

1,000 minus 650

Decrease 1,250 by 400

The difference between 2,001 and 1,995

Using related calculations

Children will use knowledge of place value and related calculations:

Use $63 - 48$ to solve $680 - 430$ and $6.3 - 4.8$

Children will continue to use the inverse relationship between addition and subtraction:

$$\square - 100 = 1,059$$

$$1,998 + \square = 2,002$$

If you know that $1,000 - 110 = 890$, what else do you know?

If you know that $0.75 + 0.25 = 1.00$, what else do you know?

Year 6

Underpinning skills (end of year expectations)

- Given a number identify 10/ 100/ 1,000/ 10,000/ 100,000/ 1,000,000 more or less
- Recognise the place value of each digit in a seven-digit whole number
- Round any number up to 10,000,000 to the nearest 10, 100, 1,000, 10,000, 100,000 or 1,000,000
- Recognise the place value of each digit in a decimal number with up to three decimal places
- Round decimal numbers with two decimal places to the nearest whole number or to one decimal place
- Derive complements of 1 e.g. $0.64 + 0.36 = 1$
- Estimate the answer to a calculation, including using the skill of rounding, and use inverse operations to check
- Derive doubles of three-digit and four-digit numbers (and decimal numbers with up to three decimal places)

Strategies

Counting on and back in steps of powers of ten (in tens, hundreds, thousands, tens of thousands, hundreds of thousands and in millions)

Children will use their understanding of place value to support counting on or back, including with the use of an empty number line:

$1,960 + 300$ count on in hundreds from 2,960

$12,250 + 260$ count on in hundreds and then tens from 12,250

$25,458 + 3,000$ count on in thousands from 25,458

$25,250 + 5,500$ count on in thousands and then hundreds from 25,250

$1,456,250 + 60,000$ count on in tens of thousands from 1,456,250

$2,256,500 + 200,000$ count on in hundreds of thousands from 2,256,500

$3,450,000 + 4,000,000$ count on in millions from 3,450,000

$1,045 - 200$ count back in hundreds from 1,045

$12,936 - 720$ count back in hundreds and then tens from 12,936

$125,856 - 235$ count back in hundreds, tens and ones from 5,856

$165,452 - 5,000$ count back in thousands from 165,452

$261,456 - 30,000$ count back in tens of thousands from 261,456

$1,857,450 - 500,000$ count back in hundred thousand from 1,857,450

$5,250,000 - 3,000,000$ count back in millions from 5,250,000

Partitioning numbers in different ways

Children will partition the second number and then add/subtract, including with the use of an empty number line:

$6,540 + 1,284 = 6,540 + 1,000 + 200 + 80 + 4$

$8,456 - 2,500 = 8,456 - 2,000 - 500$

$455,460 + 2,458 = 455,460 + 2,000 + 400 + 50 + 8$

Children use their understanding of place value to partition decimal numbers and then add/subtract:

$12.75 + 5.25 = 12.75 + 5.00 + 0.2 + 0.05$

Children will use their knowledge of number bonds and place value to partition in different ways when adding and subtracting, bridging through multiples of powers of ten:

$5,296 + 234 = 5,296 + 4 + 230$

$8,564 - 170 = 8,584 - 164 - 6$

$5.6 + 3.5 = 5.6 + 0.4 + 3.1$

Consider using an empty number line to record jottings

Re-ordering numbers when adding

Children will know that it can sometimes be easier to re-order numbers when adding:

Re-order to start with the largest number and understand the commutative property of addition

$$640 + 5,257 \text{ becomes } 5,257 + 640$$

Re-order to find pairs that total multiples of power of ten when adding/subtracting three numbers

$$1,488 + 165 + 12 \text{ becomes } 1,488 + 12 + 165 = 1,500 + 165$$

$$4.8 + 2.5 - 1.8 \text{ becomes } 4.8 - 1.8 + 2.5$$

Add and subtract multiples of 10, 100 or 1,000 and adjust

Children will use their knowledge of adding and subtracting multiples of 10, 100 or 1,000 and adjusting to add/subtract, including with the use of an empty number line:

$$845 + 28 = 845 + 30 - 2 \text{ (28 rounds up to 30)}$$

$$1,942 + 99 = 1,942 + 100 - 1 \text{ (99 rounds up to 100)}$$

$$5,856 - 198 = 5,856 - 200 + 2 \text{ (198 rounds up to 200)}$$

$$6,565 + 999 = 2,565 + 1,000 - 1 \text{ (999 rounds up to 1,000)}$$

$$8,250 - 998 = 8,250 - 1,000 + 2 \text{ (998 rounds up to 1,000)}$$

Adding near doubles

Children will use their knowledge of doubles to add near doubles:

$$2.5 + 2.6 = \text{double } 2.5 \text{ and add } 0.1$$

$$490 + 480 = \text{double } 500 \text{ and subtract } 30$$

Finding the difference by counting on

Children will use complementary addition to count on from the smaller number to the larger number to find a small difference, including with the use of an empty number line:

$$908 - 897 \text{ count up from } 897$$

$$1,015 - 998 \text{ count up from } 998$$

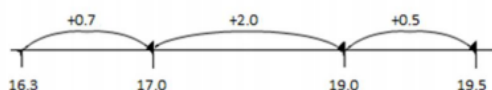
$$1,102 - 877 \text{ count up from } 877$$

$$2,017 - 1,988 \text{ count up from } 1,988$$

$$3,000 - 2,899 \text{ count up from } 2,899$$

$$10,004 - 8,997 \text{ count up from } 8,997$$

$$19.5 - 16.3 \text{ count up from } 16.3$$



Encouraging children to use number lines in this way provides a mental image that can assist with mental calculations

Rapid recall

Give children the opportunity to respond rapidly to oral and written questions, using a range of vocabulary:

$$70,000 + 30,000$$

The sum of 12,500 and 2,000

The total of 300 + 500 + 700

Add together 1.8 and 3.2

Increase 2,500 by 999

$$1,000 - 155$$

10 subtract 2.8

100 less than 10,086

1,000 minus 555

The difference between 2,001 and 1,995

Using related calculations

Children will use knowledge of place value and related calculations:

$680 + 430$, $6.8 + 4.3$, $0.68 + 0.43$ can all be worked out using the related calculation $68 + 43$

Children will continue to use the inverse relationship between addition and subtraction:

$$\square - 10,000 = 42,560$$

$$1.85 + \square = 2.00$$

If $998 + n = 1,012$ what is the value of n ?

If $1,500 - m = 600$ what is the value of m ?

If you know $0.55 + 0.45 = 1.00$, what else do you know?

Progression in multiplication and division strategies

Year 1

Underpinning skills (end of year expectation)

- Count in multiples of twos, fives and tens (to the 10th multiple)
- Understand and use the vocabulary (but **not** the signs) associated with multiplication and division in practical contexts
- Recall doubles up to double ten (10 + 10) and find the corresponding halves

Strategies

Counting

Children will count in multiples of two, five and ten, to the 10th multiple:

Use a counting stick to count forwards (and backwards)

Drop 2p coins into a jar and count in twos (then use 10p or 5p coins)

Count children when in pairs

Use counting songs and rhymes

Combining groups

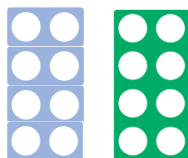
Children will combine groups of 2, 5 or ten, in practical situations:

Five pairs of socks. How many socks altogether?

2, 4, 6, 8, 10



Four groups of two is eight



Sharing and grouping

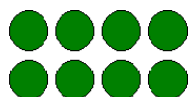
Children will share a set of objects, equally:

Share 12 apples equally between two children. How many apples will they each get? (Sharing)

There are 15 biscuits in a pack. If we put five biscuits on a plate, how many plates will we need? (Grouping)

Describing arrays

Children will develop an understanding of multiplication and division by describing and making simple arrays:



Four groups of two

Two groups of four

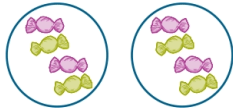
Eight counters altogether

Share eight counters equally between two children

Doubling and halving

Children will find doubles and halves, in practical situations:

Make the link between doubling and finding two groups of



Double four is eight. Two groups of four is eight

Use fingers to show doubles 'Show me double four'

Find double dominoes and describe them, making the link with addition and two groups

Double six is twelve, $6 + 6 = 12$, two groups of six is twelve



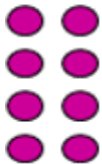
Make the link between halving and equal sharing between two

Half of twelve is six

Share twelve apples equally between two children

Make the link between doubling and halving

Double four is eight. Half of eight is four



Rapid recall

Give children the opportunity to respond rapidly to oral questions, using a range of vocabulary:

Two groups of ten

Three lots of five

How many groups of two are there in eight?

Share ten apples between two children

Double four

Half of twelve

Year 2

Underpinning skills (end of year expectation)

- Count in multiples of 2, 3, 5 and 10 from 0, forwards and backwards (to the 12th multiple)
- Recall multiplication/division facts for the 2, 5 and 10 times table to the 12th multiple
- Read, write and interpret mathematical statements involving multiplication and division, including the signs \times , \div and $=$, and understand and use the associated vocabulary
- Know that multiplication of two numbers can be done in any order (commutative) but that division of one number by another cannot
- Recall the doubles of multiples of 10 to 100 (e.g. double 40 is 80) and recall the related halves (e.g. half of 80 is 40)

Strategies

Counting

Children will count in multiples of two, **three**, five and ten, to the 12th multiple:

Use a counting stick to count forwards (and backwards)

Ask children to count from zero in a known multiple e.g. fives. When you clap, they count backwards. On the next clap, they count forwards, and so on...

Drop 2p coins into a jar and count in twos (then use 10p and 5p coins)

Count around the clock in fives

Use counting songs and rhymes

Combining groups

Children will count groups of two, three, five and ten:

Five apples in a bag. How many apples in four bags?

5, 10, 15, 20

Multiplication as repeated addition

Children will represent multiplication as repeated addition

Four groups of five

5, 10, 15, 20

$$5 + 5 + 5 + 5 = 20$$

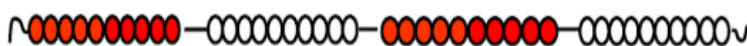
$$4 \times 5 = 20$$

Four groups of ten

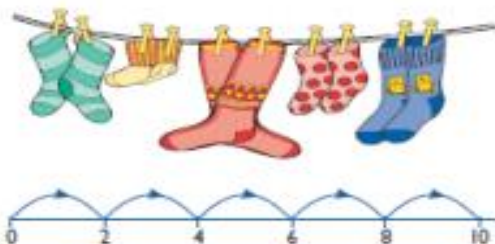
10, 20, 30, 40

$$10 + 10 + 10 + 10 = 40$$

$$4 \times 10 = 40$$



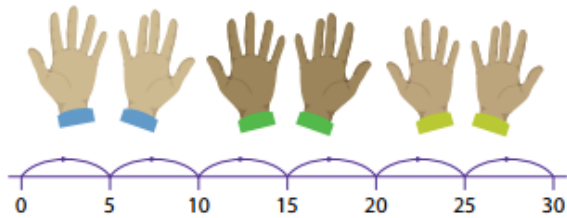
Children can also begin to use empty number lines to count on in groups (multiples) of 2, 3, 5 and 10



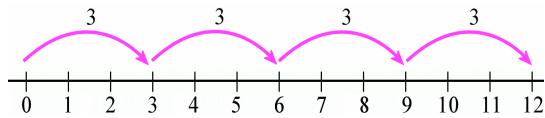
2, 4, 6, 8, 10

Five jumps of two

$$2 + 2 + 2 + 2 + 2 = 5 \times 2 = 10$$



Six groups of five
 0, 5, 10, 15, 20, 25, 30
 $5 + 5 + 5 + 5 + 5 + 5$
 $6 \times 5 = 30$



Four groups of three, four jumps of three
 0, 3, 6, 9, 12
 $3 + 3 + 3 + 3 = 12$
 $4 \times 3 = 12$

Sharing and grouping

Children will move from sharing to grouping:

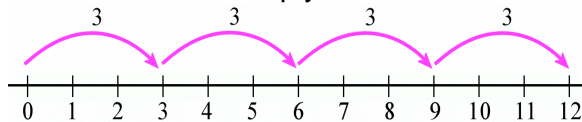
Twenty apples are shared equally between five children. How many apples will they each have? (Sharing)

I have 20 apples and I want to put them into bags of five. How many bags do I need?

5, 10, 15, 20 (Grouping)

$$20 \div 5 = 4$$

Children can use empty number lines to count on, to make the link with multiplication



How many groups of three are there in twelve?

$$12 \div 3 = 4$$

Counting back on a number line makes the link with repeated subtraction

$$12 - 3 - 3 - 3 - 3$$

Arrays

Children will further develop an understanding of multiplication by describing and making arrays:

Four groups of five

$$5 + 5 + 5 + 5 = 20$$

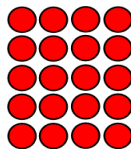
$$4 \times 5 = 20$$

This can also be described as

Five groups of four

$$4 + 4 + 4 + 4 + 4 = 20$$

$$5 \times 4 = 20$$



By making arrays children will see that multiplication can be done in any order (multiplication is commutative)

$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

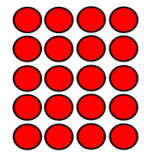
Arrays can also be used to support an understanding of division

20 counters altogether. How many groups of five are there?

$$20 \div 5 = 4$$

How many groups of four are there?

$$20 \div 4 = 5$$



Children can count on (count forwards), to make the link with multiplication

How many fives are there in twenty? 5, 10, 15, 20 (four groups of five)

Children can make their own arrays with counters, describing them using the language of multiplication and division

Doubling and halving

Children will find doubles and related halves of numbers making the link with multiplying and dividing by two:

Double 12 is 24.

Two groups of 12 is 24

$$12 \times 2 = 24$$

Half of 24 is 12

Share 24 equally between two

$$24 \div 2 = 12$$



Rapid recall

Give children the opportunity to respond rapidly to oral and written questions, using a range of vocabulary:

Four groups of five

$$6 \times 10$$

Eight times two

Five multiplied by three

$$3 \times 0$$

Double 40

How many twos in 16?

How many groups of five are there in twenty?

Divide 40 by 10

$$30 \div 5$$

Half of 80

Using related calculations

Children will use knowledge of place value, inverse operations and related calculations:

$3 \times 5 = 15$ therefore...

$$5 \times 3 = 15$$

$$15 \div 5 = 3$$

$$15 \div 3 = 5$$

$$3 \times \square = 15$$

$$\square \div 5 = 3$$



Double 4 is 8 therefore ...

Double 40 is 80

Half of 80 is 40

Year 3

Underpinning skills (end of year expectation)

- Count in multiples of 2, 3, 4, 5, 8, 10, 50 and 100 from 0, forwards and backwards (to the 12th multiple)
- Recall multiplication/division facts for the 2, 3, 4, 5, 8 and 10 times tables to the 12th multiple
- Understand and use the vocabulary and signs associated with multiplication and division
- Understand the effect of multiplying/dividing numbers by 10
- Understand the commutative properties of multiplication
- Recognise the inverse relationship between multiplication and division
- Derive doubles of all two-digit numbers (e.g. double 42 is 84) and the corresponding halves (half of 84 is 42)

Strategies

Counting

Children will count in multiples of 2, 3, 4, 5, 8, 10, 50 and 100 to the 12th multiple:

Use a counting stick to count forwards (and backwards) asking related multiplication and division questions

Ask children to count from zero in a known multiple e.g. fours. When you clap, they count backwards. On the next clap, they count forwards, and so on...

Count around the clock in fives

Play Fizz, buzz with multiples of three and five

Use counting songs and rhymes

Arrays

Children will further develop an understanding of multiplication and division by describing and making arrays:



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$

$$18 \div 3 = 6$$

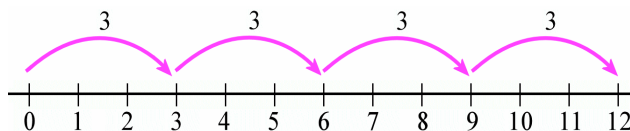
$$18 \div 6 = 3$$

Discuss the commutative property of multiplication and that multiplication and division are inverse operations

Give children 12 (or 24) counters and ask them to make an array, then describe their array using multiplication and division facts

Using empty number lines

Children can also use empty number lines to count on in multiples of 2, 3, 4, 5, 8 and 10



0, 3, 6, 9, 12

Four groups of three

Four jumps of three

$$4 \times 3 = 12$$

Children can use empty number lines to record division, counting on/forwards to make the link with multiplication

How many threes are in there in twelve?

$$12 \div 3 = 4$$

Counting back on a number line makes the link with repeated subtraction

$$12 - 3 - 3 - 3 - 3 = 0$$

Remainders can be modelled with arrays and/or with empty number lines

Using partitioning to multiply and divide

Children will multiply teen number by a known multiple using their knowledge of place value:

14×5 (partition 14 into $10 + 4$)

$$10 \times 5 = 50$$

$$4 \times 5 = 20$$

$$50 + 20 = 70$$

$$14 \times 5 = 70$$

Consider the use of base ten resources to support

This can also be recorded using a grid

X	10	4
5	50	20

Add the partial products together

$$50 + 20 = 70$$

$$14 \times 5 = 70$$

Children can also **begin** to use partitioning to divide

$42 \div 3$ (partition 42 into 30 and 12)

$$30 \div 3 = 10$$

$$12 \div 3 = 4$$

$$10 + 4 = 14$$

$$42 \div 3 = 14$$

Multiplying and dividing by ten and multiples of ten

Children will use their understanding of place value to multiply by ten and multiples of ten:

$$3 \times 10 = 30$$

$$30 \div 10 = 3$$



Consider using a rectangular array to model multiplication by ten

$$7 \times 10 = 70$$

$$70 \times 10 = 700$$

Multiply by ten by shifting digits one place to the left and placing zero in the ones/units column as a place holder

$$70 \div 10 = 7$$

$$700 \div 10 = 70$$

Divide by ten by shifting digits one place to the right

Consider using a place value chart to support understanding of multiplying and dividing by ten

$$24 \times 10 = 240$$

$$240 \div 10 = 24$$

Extend with multiplying by other multiples of ten

$$3 \times 20 = 3 \times 10 \times 2 = 60$$



$$4 \times 30 = 4 \times 3 \times 10 = 120$$

Doubling and halving

Children will find doubles and related halves of numbers making the link with multiplying and dividing by two:

Double 24 is 48

$$2 \times 24 = 48$$

Half of 48 is 24

$$48 \div 2 = 24$$

Children can use partitioning to support finding doubles of two-digit numbers

Double 38 (partition 38 into 30 + 8)

$$\text{Double } 30 = 60$$

$$\text{Double } 8 = 16$$

$$60 + 16 = 76$$

$$\text{Double } 38 = 76$$

$$\text{Half of } 76 = 38$$

Rapid recall

Give children the opportunity to respond rapidly to oral and written questions, using a range of vocabulary:

$$6 \times 4$$

8 multiplied by 5

$$4 \times 0$$

Double 60

$$36 \div 4$$

$$8 \div 8$$

How many threes 'go into' 27?

Half of 120

Using related calculations

Children will use knowledge of place value, inverse operations and related calculations:

$$8 \times 5 = 40 \text{ therefore...}$$

$$5 \times 8 = 40$$

$$40 \div 5 = 8$$

$$40 \div 8 = 5$$

$$8 \times \square = 32$$

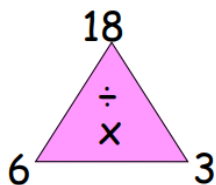
$$\square \div 6 = 3$$

$$3 \times 4 = 12 \text{ therefore } 3 \times 40 = 120, 30 \times 4 = 120$$

Double 25 is 50. Therefore double 250 is 500

Half of 50 is 25. Therefore half of 500 is 250

Write four facts using this trio of numbers



If you know $8 \times 4 = 32$, what else do you know?

Year 4

Underpinning skills (end of year expectation)

- Count in multiples of 2, 3, 4, 5, **6, 7, 8, 9, 10, 11, 12, 25**, 50, 100 and 1000 from 0, forwards and backwards (to the 12th multiple)
- Recall multiplication and division facts for multiplication tables up to 12 x 12
- Understand the effect of multiplying by 0 or 1 and dividing by 1
- Recognise and identify factor pairs
- Understand the effect of multiplying/dividing numbers by 10/100, including decimal numbers
- Recall doubles of two-digit numbers and derive doubles of three-digit numbers and find the corresponding halves
- Estimate the answer to a calculation, including the use of rounding, and use inverse operations to check

Strategies

Counting

Count in multiples of 2, 3, 4, 5, **6, 7, 8, 9**, 10, **11, 12, 25**, 50, 100 and 1000 from 0, forwards and backwards (to the 12th multiple):

Use a counting stick to count forwards (and backwards) asking related multiplication and division questions

Ask children to count from zero in a known multiple e.g. sixes. When you clap, they count backwards. On the next clap, they count forwards, and so on.....

Play Fizz, buzz with multiples of three and five (or multiples of four and six)

Multiplying and dividing by 10/100

Children will use their understanding of place value to multiply/divide by ten and multiples of ten:

$$9 \times 10 = 90$$

$$90 \times 10 = 900$$

Multiply by ten by shifting digits one place to the left and placing zero in the ones/units column as a place holder

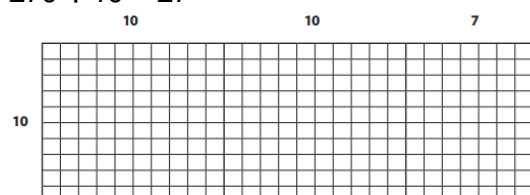
$$90 \div 10 = 9$$

$$900 \div 10 = 90$$

Divide by ten by shifting digits one place to the right

$$27 \times 10 = 270$$

$$270 \div 10 = 27$$



This area model shows how a two-digit number has been partitioned into tens and ones and multiplied by 10. Children can then visualise this at a later stage to aid mental calculation

$$54 \times 10 = 540$$

$$540 \div 10 = 54$$

Multiply and divide by multiples of ten

$$6 \times 30 = 6 \times 3 \times 10 = 18 \times 10 = 180$$

$$180 \div 30 = 180 \div 10 \div 3 = 18 \div 3 = 6$$

Extend with decimal numbers

$$2.4 \times 10 = 24$$

$$24 \div 10 = 2.4$$

Children will use their understanding of place value to multiply/divide by one hundred:

$$4 \times 100 = 400$$

$$400 \div 100 = 4$$

$$35 \times 100 = 3,500$$

$$3,500 \div 100 = 35$$

Multiply by one hundred by shifting digits two places to the left and placing zero in the ones/units column as a place holder

Divide by one hundred by shifting digits two places to the right

Extend with decimal numbers (with one decimal place)

$$2.4 \times 100 = 240$$

$$240 \div 100 = 2.4$$

Consider using a place value chart to support understanding of multiplying and dividing numbers by 10/100

Using partitioning and the distributive law to multiply

Children will multiply a two-digit number by a known multiple using their understanding of place value:

$$16 \times 5 = (10 \times 5) + (6 \times 5)$$

$$= 50 + 30$$

$$= 80$$

$$32 \times 3 = (30 \times 3) + (2 \times 3)$$

$$= 90 + 6$$

$$= 96$$

This can also be recorded using a grid

X	30	2
3	90	6

Add the partial products together

$$24 \times 7 = (20 \times 7) + (4 \times 7)$$

$$= 140 + 28$$

$$= 168$$

X	20	4
7	140	28

Add the partial products together

Using partitioning to divide

Children will use their knowledge of partitioning numbers in different ways to divide a two-digit number by a single-digit number:

$$48 \div 3 \text{ (partition 48 into 30 and 18)}$$

$$30 \div 3 = 10$$

$$18 \div 3 = 6$$

$$10 + 6 = 16$$

$$48 \div 3 = 16$$

Extend by simplifying the recording

$$78 \div 6 = (60 \div 6) + (18 \div 6)$$

$$10 + 3 = 13$$

$$78 \div 6 = 13$$

Using multiples and factor pairs

Children will begin to recognise and use factor pairs to aid multiplication:

$$7 \times 20 = 7 \times 2 \times 10 = 14 \times 10$$

$$6 \times 15 = 6 \times 5 \times 3 = 30 \times 3$$

$$4 \times 24 = 4 \times 2 \times 12 = 8 \times 12$$

Children will use their knowledge of multiples, factors and their understanding that multiplication can be done in any order (multiplication is commutative) to multiply three numbers together:

$$2 \times 6 \times 5 = 2 \times 5 \times 6 = 10 \times 6$$

$$3 \times 7 \times 4 = 3 \times 4 \times 7 = 12 \times 7$$

Doubling and halving

Children will find doubles and related halves of numbers making the link with multiplying and dividing by two:

Double 75 is 150

$$2 \times 75 = 150$$

Half of 150 is 75

$$150 \div 2 = 75$$

Children can use partitioning to support finding doubles and halves of two-digit and three-digit numbers:

Double 86 (partition 86 into 80 + 6)

$$\text{Double } 80 = 160$$

$$\text{Double } 6 = 12$$

$$160 + 12 = 172$$

Double 86 is 172

Double 248 (partition 248 into 200 + 40 + 8)

$$\text{Double } 200 = 400$$

$$\text{Double } 40 = 80$$

$$\text{Double } 8 = 16$$

$$400 + 80 + 16 = 496$$

Double 248 is 496

Half of 632 (partition 632 into 600 and 32)

$$\text{Half of } 600 = 300$$

$$\text{Half of } 32 = 16$$

$$300 + 16 = 316$$

Half of 632 is 316

Rapid recall

Give children the opportunity to respond rapidly to oral and written questions using a range of vocabulary:

$$8 \times 6$$

7 multiplied by 5

What is the product of 9 and 10?

Multiply three by twelve

$$7 \times 0$$

Double 64

Divide 28 by 4

$$36 \div 9$$

How many sixes 'go into' 42?

$$12 \div 12$$

77 divided by 11

Half of 420

Using related calculations

Children will use knowledge of place value, inverse operations and related calculations:



Write four facts using this trio of numbers

$$12 \times 6 = 72$$

$$6 \times 12 = 72$$

$$72 \div 6 = 12$$

$$72 \div 12 = 6$$

$8 \times 6 = 48$ therefore $8 \times 60 = 480$, $80 \times 6 = 480$, $8 \times 600 = 4,800 \dots$

If you know $6 \times 8 = 48$, what else do you know?

Double 75 is 150. Therefore double 750 is 1,500

14×5 becomes 7×10 (halve 14 and double 5)

Derive the 6x table facts by doubling the 3x table facts; derive the 12x table facts by doubling the 6x table facts

Year 5

Underpinning skills (end of year expectation)

- Count in multiples of 3, 4, 6, 7, 8, 9, 11, 12, 25, 50, 100 and 1,000 forwards and backwards
- Consolidate multiplication and division facts for multiplication tables up to 12×12
- Find **all** factor pairs of a given number; find **all** common factors for a pair of numbers; identify multiples
- Derive all square numbers to 12^2 ($12 \times 12 = 144$)
- Understand the effect of multiplying/dividing whole numbers, and decimal numbers with up to two decimal places, by 10, by 100 and by 1,000
- Derive doubles of three-digit and four-digit numbers (and decimal numbers with up to two decimal places) and find the corresponding halves
- Estimate the answer to a calculation and use inverse operations to check

Strategies

Counting

Count in multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 25, 50, 100 and 1000 from 0, forwards and backwards (to the 12th multiple):

Use a counting stick to count forwards (and backwards) using known multiples, asking related multiplication and division questions; extend by counting in other multiples e.g. multiples of 40 or multiples of 0.4, using knowledge of place value

Ask children to count from zero in a known multiple e.g. 25s. When you clap, they count backwards. On the next clap, they count forwards, and so on...

Multiplying and dividing by 10/100/1,000

Children will use their understanding of place value to multiply/divide by ten and multiples of ten:

Multiply by ten by shifting digits one place to the left (and placing zero in the ones/units column as a place holder, when appropriate)

$$0.9 \times 10 = 9$$

$$9 \times 10 = 90$$

$$90 \times 10 = 900$$

$$900 \times 10 = 9,000$$

$$4.5 \times 10 = 45$$

$$45 \times 10 = 450$$

$$450 \times 10 = 4,500$$

$$45 \times 20 = 45 \times 2 \times 10 = 900$$

Divide by ten by shifting digits one place to the right

$$9 \div 10 = 0.9$$

$$90 \div 10 = 9$$

$$900 \div 10 = 90$$

$$9,000 \div 10 = 900$$

$$45 \div 10 = 4.5$$

$$450 \div 10 = 45$$

$$4,500 \div 10 = 450$$

$$900 \div 20 = 900 \div 10 \div 2 = 45$$

Children will use their understanding of place value to multiply/divide by one hundred and multiples of 100:

Multiply by one hundred by shifting digits two places to the left (and placing zero in the ones/units column as a place holder, when appropriate)

Divide by one hundred by shifting digits two places to the right

$$0.4 \times 100 = 40$$

$$40 \div 100 = 0.4$$

$$40 \times 100 = 4,000$$

$$4,000 \div 100 = 40$$

$$2.45 \times 100 = 245$$

$$245 \div 100 = 2.45$$

$$24 \times 200 = 24 \times 2 \times 100 = 48 \times 100 = 4,800$$

$$4,800 \div 200 = 4,800 \div 100 \div 2 = 48 \div 2 = 24$$

Children will use their understanding of place value to multiply/divide by one thousand:

Multiply by one thousand by shifting digits three places to the left (and placing zero in the ones/units column as a place holder, when appropriate)

Divide by one thousand by shifting digits three places to the right

$$62 \times 1,000 = 62,000$$

$$62,000 \div 1,000 = 62$$

$$2.5 \times 1,000 = 2,500$$

$$2,500 \div 1,000 = 2.5$$

$$0.45 \times 1,000 = 450$$

$$450 \div 1,000 = 0.45$$

Consider using a place value chart to support understanding of multiplying and dividing numbers by 10/100/ 1000

Using partitioning and the distributive law to multiply

Children will multiply a two-digit number by a known multiple using their understanding of place value:

$$36 \times 7 = (30 \times 7) + (6 \times 7)$$

$$= 210 + 42$$

$$= 252$$

$$47 \times 8 = (40 \times 8) + (7 \times 8)$$

$$= 320 + 56$$

$$= 376$$

Using partitioning to divide

Children will use their knowledge of partitioning numbers in different ways to divide a two-digit number by a single-digit number, including answers with remainders:

$$84 \div 6 = (60 \div 6) + (24 \div 6)$$

$$60 \div 6 = 10$$

$$24 \div 6 = 4$$

$$84 \div 6 = 14$$

$$87 \div 5 = (50 \div 5) + (37 \div 5)$$

$$50 \div 5 = 10$$

$$37 \div 5 = 7 \text{ remainder } 2$$

$$87 \div 5 = 17 \text{ remainder } 2 \text{ (or } 17 \frac{2}{5})$$

Extend with three-digit numbers divided by a single digit number:

$$132 \div 6 = (120 \div 6) + (12 \div 6)$$

$$20 + 2 = 22$$

$$132 \div 6 = 22$$

Using factor pairs

Children will recognise and use factor pairs to aid multiplication and division:

$$8 \times 16 = 8 \times 8 \times 2 = 64 \times 2 = 128$$

$$12 \times 14 = 12 \times 7 \times 2 = 84 \times 2 = 168$$

$$25 \times 12 = 25 \times 4 \times 3 = 100 \times 3 = 300$$

$$90 \div 6 = (90 \div 3) \div 2 = 30 \div 2 = 15$$

$$120 \div 8 = (120 \div 4) \div 2 = 30 \div 2 = 15$$

Doubling and halving

Children will know or derive doubles and related halves of numbers:

Double 75 is 150, half of 150 is 75

Double 7.5 is 15, half of 15 is 7.5

Double 0.75 is 1.5, half of 1.5 is 0.75

Children can use partitioning to support finding doubles of two-digit and three-digit numbers:

Double 176 (partition 176 into 100 + 70 + 6)

Double 100 = 200

Double 70 = 140

Double 6 = 12

$$200 + 140 + 12 = 352$$

Double 176 is 352

Half of 256 (partition 256 into 200 + 50 + 6)

Half of 200 = 100

Half of 50 = 25

Half of 6 = 3

$$100 + 25 + 3 = 128$$

Half of 256 is 128

Rapid recall

Give children the opportunity to respond rapidly to oral and written questions using a range of vocabulary:

$$8 \times 7$$

7 multiplied by 3

Multiply 7 by 9

What is the product of 9 and 6?

Double 135

What is six squared?

$$9 \times 0$$

$$8^2$$

$$45 \div 9$$

Divide 56 by seven

How many twelves 'go into' 72?

64 divided by 8

Divide 96 by 12

What is the quotient when you divide 63 by 7?

$$144 \div 144$$

Half of 428

Using related calculations

Children will use knowledge of place value, inverse operations and related calculations:

$$12 \times 8 = 96 \text{ therefore...}$$

$$8 \times 12 = 96$$

$$96 \div 12 = 8$$

$$96 \div 8 = 12$$

$$8 \times \square = 9.6$$

$$960 \div \square = 12$$

If you know $12 \times 8 = 96$, what else do you know?

46×5 becomes 23×10 (halve 46 and double 5)

35×14 becomes 70×7 (double 35 and halve 14)

75×4 can be found by doubling and doubling again

Year 6

Underpinning skills (end of year expectation)

- Recall multiplication/division facts for all multiplication tables up to 12×12 **with fluency**
- Identify factors, common factors, common multiples and prime factors
- Recall all square numbers to 12^2 ($12 \times 12 = 144$)
- Understand the effect of multiplying/dividing whole numbers, and decimal numbers with up to three decimal places, by 10, by 100 and by 1,000
- Understand the order of operations using brackets (BODMAS)
- Estimate the answer to a calculation and use inverse operations to check

NB Ensure that underpinning skills, knowledge and strategies from previous year groups are secure

Strategies

Counting

Count in multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 25, 50, 100 and 1000 from 0, forwards and backwards (to the 12th multiple):

Use a counting stick to count forwards (and backwards) using known multiples, asking related multiplication and division questions; extend by counting in other multiples e.g. multiples of 70 or multiples of 0.7, using knowledge of place value

Ask children to count from zero in a known multiple e.g. 25s. When you clap, they count backwards. On the next clap, they count forwards, and so on; extend by counting in multiples of 250 or 2.5

Multiplying and dividing by 10/100/1,000

Children will use their understanding of place value to multiply/divide by ten and multiples of ten:

Multiply by ten by shifting digits one place to the left (and placing zero in the ones/units column as a place holder, when appropriate)

$$6.5 \times 10 = 65$$

$$65 \times 10 = 650$$

$$650 \times 10 = 6,500$$

$$65 \times 20 = (65 \times 10) \times 2 = 1,300$$

Divide by ten by shifting digits one place to the right

$$65 \div 10 = 6.5$$

$$650 \div 10 = 65$$

$$6,500 \div 10 = 650$$

$$1,300 \div 20 = (1,300 \div 10) \div 2$$

Children will use their understanding of place value to multiply/divide by one hundred and multiples of 100:

Multiply by one hundred by shifting digits two places to the left (and placing zero in the ones/units column as a place holder, when appropriate)

Divide by one hundred by shifting digits two places to the right

$$2.05 \times 100 = 205$$

$$205 \div 100 = 2.05$$

$$2.5 \times 300 = (2.5 \times 100) \times 3 = 250 \times 3 = 750$$

$$750 \div 300 = (750 \div 100) \div 3 = 7.5 \div 3 = 2.5$$

Children will use their understanding of place value to multiply/divide by one thousand and **extend** with multiples of 1,000:

Multiply by one thousand by shifting digits three places to the left (and placing zero in the ones/units column as a place holder, when appropriate)

Divide by one thousand by shifting digits three places to the right

$$162 \times 1,000 = 162,000$$

$$162,000 \div 1,000 = 162$$

$$7.5 \times 1,000 = 7,500$$

$$7,500 \div 1,000 = 7.5$$

$$0.25 \times 1,000 = 250$$

$$250 \div 1,000 = 0.25$$

$$48 \times 2,000 = (48 \times 1,000) \times 2 = 48,000 \times 2 = 96,000$$

$$96,000 \div 2,000 = (96,000 \div 1,000) \div 2 = 48$$

Consider using a place value chart to support understanding of multiplying and dividing numbers by 10/100/ 1000

Using partitioning and the distributive law to multiply

Children will multiply a two-digit number by a known multiple using their understanding of place value:

$$\begin{aligned} 46 \times 7 &= (40 \times 7) + (6 \times 7) \\ &= 280 + 42 \\ &= 322 \end{aligned}$$

$$\begin{aligned} 2.6 \times 8 &= (2 \times 8) + (0.6 \times 8) \\ &= 16 + 4.8 \\ &= 20.8 \end{aligned}$$

Using partitioning to divide

Children will use their knowledge of partitioning numbers in different ways to divide a two-digit number or a three-digit number by a single-digit number, including answers with remainders:

$$85 \div 5 = (50 \div 5) + (35 \div 5)$$

$$10 + 7 = 17$$

$$85 \div 5 = 17$$

$$97 \div 6 = (60 \div 6) + (37 \div 6)$$

$$60 \div 6 = 10$$

$$37 \div 6 = 6 \text{ remainder } 1$$

$$97 \div 6 = 16 \text{ remainder } 1 \text{ (or } 16\frac{1}{6}\text{)}$$

$$161 \div 7 = (140 \div 7) + (21 \div 7)$$

$$140 \div 7 = 20$$

$$21 \div 7 = 3$$

$$154 \div 7 = 22$$

Using factor pairs

Children will recognise and use factor pairs to aid multiplication and division:

$$9 \times 18 = 9 \times 9 \times 2 = 81 \times 2 = 162$$

$$25 \times 16 = 25 \times 4 \times 4 = 100 \times 4 = 400$$

$$35 \times 18 = 35 \times 2 \times 9 = 70 \times 9 = 630$$

$$150 \div 6 = (150 \div 3) \div 2 = 50 \div 2 = 25$$

$$210 \div 14 = (210 \div 7) \div 2 = 30 \div 2 = 15$$

Doubling and halving

Children will know or derive doubles and related halves of numbers:

Use the fact double 85 is 170 to derive...

Half of 170; double 8.5; half of 17; double 0.85; half of 1.7

Children can use partitioning to support finding doubles of two-digit and three-digit numbers, including decimal numbers:

Double 387 (partition 387 into 300 + 80 + 7)

Double 387 = 600 + 160 + 14 = 774

Half of 984 (partition 984 into 900 + 80 + 4)

Half of 984 = 450 + 40 + 2 = 492

(Use inverse to check)

Half of £71.30 (partition £71.30 into £70 + £1.00 + £0.30)

Half of £71.30 = £35 + +£0.50 + £0.15 =£35.65

(Use the inverse operation inverse to check)

Rapid recall

Give children the opportunity to respond rapidly to oral and written questions, using a range of vocabulary:

8×7

70 multiplied by 3

What is the product of 9 and 8?

25×4

Double 258

What is twelve squared?

9^2

What is two cubed?

10^3

$72 \div 9$

Divide 56 by seven

What is the quotient when 132 is divided by 12

How many twelves 'go into' 96?

$200 \div 25$

Half of 1,500

Using related calculations

Children will use knowledge of place value, inverse operations and related calculations:

$$9 \times \square = 6.3$$

$$630 \div \square = 9$$

$$6 \times a = 72, a = ?$$

$36 = a \times b$ what are the possible values of a and b

If you know $9 \times 7 = 63$, what else do you know?

46×50 becomes 23×100 (halve 46 and double 50)

45×14 becomes 90×7 (double 45 and halve 14)

How does $9 \times 12 = 108$ help you to calculate 18×6 ?

$$13 \times 99 = (13 \times 100) - 99$$

125×4 can be found by doubling and doubling again

$500 \div 4$ can be found by halving and halving again

NOTES